→ Sets :-

- A Set is a collection of well-defined objects. ~~is~~
- Sets are usually derived / denoted as : A, B, C, D ---

eg:- • The set of all days in a week.
• The set of natural no less than 9.

↳ Representation of Sets :-

- There are 2 ways of representing a set :

(i) Tabular / Roster method :-

• In this, we write all the elements of a set within 2 flower brackets & elements are seperated by commas (,)

eg:- $A = \{4, 7, 1\}$

$B = \{a, b, d, e\}$

(ii) Rule / Set builder method :-

• In this, we specify the set by stating the common property that every element of the set is satisfied.

eg:- • $A = \{x : x$ is an even natural number less than $10\}$

$A = \{2, 4, 6, 8\}$

• $B = \{x : x$ is a letter of the word MAT

$= \{M, A, T, H, E, I, C, S\}$

Eg. $C = \{x : x^2 - 16 = 0\}$ $\quad C = \{-4, 4\}$

$\because x^2 - 16 = 0$

$\Rightarrow x^2 - 16$

$x = \sqrt{16}$

$\therefore \boxed{x = \pm 4}$

Eg. $D = \{x : x^2 + 5x + 6 = 0\}$

$D = \{-3, -2\}$

- $x^2 + 5x + 6 = 0$

$x^2 + 3x + 2x + 6 = 0$

$x(x+3) + 2(x+3) = 0$

$\Rightarrow (x+3)(x+2) = 0$

$x + 3 = 0$ (or) $x + 2 = 0$

$\Rightarrow x = -3$ (or) $x = -2$

- $x^2 + 5x + 6 = 0$

$$\boxed{x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}}$$

$a = 1$
$b = 5$
$c = 6$

$$= \frac{-5 \pm \sqrt{25 - 4(1)(6)}}{2(1)}$$

$$= \frac{-5 \pm \sqrt{1}}{2}$$

$$= \frac{-5 \pm 1}{2}$$

$$= \frac{-5+1}{2} \text{ (or) } \frac{-5-1}{2}$$

$$= -2 \text{ (or) } -3$$

Note:- If any element is repeated in set, then it is counted only once.

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→ Types of sets :-

(i) Null / Empty set :-

- A set consisting of no elements is called null/empty set.
- A null set is denoted by symbol ϕ (or) { }.

eg:- $A = \{x : x$ is a natural number, $2 < x < 3\}$

$A = \phi$

(ii) Single-ton set :-

- A set which has only one element is called singleton set.

eg:- $A = \{a\}$

(iii) Equal set :-

- 2 sets A & B are said to be equal if they have a same elements.
- i.e, each element of A is also a element of B & each element of B is also a element of A.
- We denote this by $A = B$.

eg:- $A = \{x : x$ is a letter of word WOLF$\}$

$B = \{x : x$ is a letter of word FOLLOW$\}$

$A = \{W, O, L, F\}$
$B = \{F, O, L, W\}$

$A = B$

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(iv) Finite & Infinite set :-

- A finite sets are sets having a finite / countable no of elements, otherwise it is infinite set.

eg:- $A = \{1, 2, 5, 7\} \rightarrow$ Finite set.

* Some important infinite set.

① Set of natural nos :

$N = \{1, 2, 3, --\}$

② Set of whole nos :

$W = \{0, 1, 2, 3, --\}$

③ Set of integers :

I or $Z = \{0, \pm 1, \pm 2, --\}$

④ Set of rational no :

$Q = \{\frac{p}{q} : p, q \in Z, q \neq 0\}$

⑤ Set of Irrational no :

$Ir = \{-- \sqrt{2}, \sqrt{3}, \sqrt{7}, e^3, \log(4) --\}$

⑥ Set of real nos :

$R =$ All rational & irrational nos.

⑦ Set of complex nos :

$C = \{x + iy : x, y \in R \quad i = \sqrt{-1}\}$

(v) Subset :- Let A & B are 2 non-empty sets,

- Set 'A' is said to be subset of 'B', if every element of 'A' is also element of 'B'.

eg:- $A = \{1, 2, 3\}$ $B = \{1, 2, 3, 4, 5, 6\}$

$A \subseteq B$, $B \supset A$, $A \subset B$

∴ clearly every element of 'A' is subset of 'B' & 'A' is also element of 'B'.

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eg:- $A = \{a, b\}$ $B = \{a, b\}$

$A \subseteq B$, $B \subseteq A$

$A = B$

- If $A \subset B$, then $x \in A \Rightarrow x \in B$

Note :-

- If 'A' is subset of 'B', then 'B' is called superset of 'A'
- Suppose $A \subseteq B$ & $B \subseteq A$, then $A = B$.
- Every set is subset of itself.
- If $A \subseteq B$ & $A \neq B$, then 'A' is called proper subset of 'B'.
- Nullset is subset of every set.

(vi) Universal set :- A universal set is a set which is containing sets / all elements under consideration.

- Universal set is always denoted by 'U'.

eg:- $U = \{1, 2, 3, 4, 5, 6\}$

$A = \{1, 2\}$ $B = \{4, 6\}$

$A \subset U$, $B \subset U$

(vii) Power set :- Let 'A' be any set, the set of all subsets of 'A' is called power set of 'A' & is denoted by P(A)

eg:- $A = \{a, b, c\}$ $n(A) = 3$ 2^3

$P(A) = \{\underset{1}{\{a\}}, \underset{2}{\{b\}}, \underset{3}{\{c\}}, \underset{4}{\{a, b\}}, \underset{5}{\{b, c\}}, \underset{6}{\{a, c\}}, \underset{7}{A}, \underset{8}{\phi}\}$

$n[P(A)] = 8$

Note :-

- If 'A' has 'n' no of elements, then Powerset of 'A' has 2^n no of elements.

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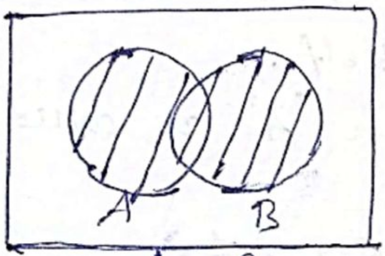
Operations on sets:-

① **Union of sets:-** Let A & B be 2 sets, the union of A & B is denoted by $A \cup B$ which is set of all elements belongs to set (A) or belongs to set (B) or to both.

$$A \cup B = \{x : x \in A \text{ or } x \in B\}$$



$A \cup B$

Note:-

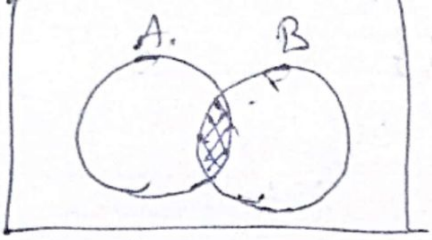
1) If $x \in A \cup B \Rightarrow x \in A$ (or) $x \in B$
2) If $x \notin A \cup B \Rightarrow x \in A$ (&) $x \notin B$

Eg:- $A = \{2, 3, 4\}$
$B = \{5, 8\}$
$A \cup B = \{2, 3, 4, 5, 8\}$

$A = \phi$, $B = \{a, b\}$
$A \cup B = \{a, b\}$

3) If $A \subseteq A \cup B$, then $B \subseteq A \cup B$.
4) If $A \subseteq B$, then $A \cup B = B$
5) If 'A' is any set, then and U is the corresponding universal set.
$A \cup U = U$
6) For any set K , $A \cup \phi = A$

② **Intersection of set:-** Let A & B are any two sets, the intersection of set A & B is denoted by $A \cap B$ is the set of all elements which belong to both A & B.



$A \cap B$

- If $x \in A \cap B$, then $x \in A$ & $x \in B$

  $x \notin A \cap B$, then $x \notin A$ or $x \notin B$

Eg:- $A = \{2, 4, 5\}$ $B = \{3, 4, 8\}$

$A \cap B = \{4\}$

- $A = \{2, 4, 5\}$ $B = \{a, b\}$

  $A \cap B = \{\phi\}$

- $A = \phi$ $B = \{a, b\}$

  $A \cap B = \phi$

Note:-

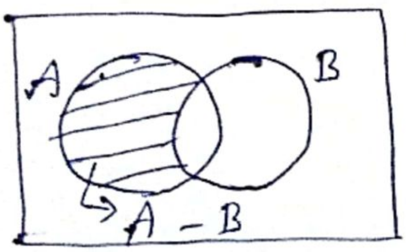
- If $A \subset B$ then $A \subset B = A$
- $A \subseteq A \cap B$, then $B \subseteq A \cap B$
- If 'A' is any set & U is the corresponding universal set, then $A \cap U = A$

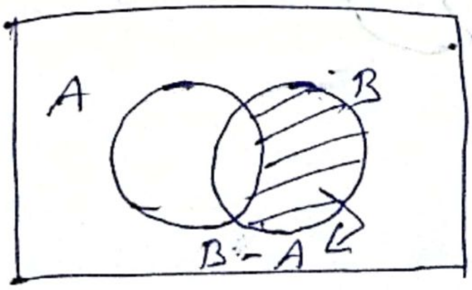
(3) Difference of sets:- Let A & B are any two sets & the set of all element of 'A' which are not in 'B' is called the difference;

- A to B denoted $A - B$; so that is:

$A - B = \{x : x \in A$ but $x \notin B\}$

$B - A = \{x : x \in B$ but $x \in A\}$





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Eg:- $A = \{1, 2, 5\}$ $\quad B = \{2, 4, 6\}$

$A - B = \{1, 5\}$

$B - A = \{4, 6\}$

- $A = \phi$, $B = \{2, 4\}$

  $A - B = \phi$

  $B - A = \{2, 4\}$

- $A = \{1, 2\}$ $\quad B = \{1, 2, 8\}$

  $A - B = \phi$

  $B - A = \{8\}$

Note:-

- If $A \subseteq B$, then $A - B = \phi$.
- If $A - B$ is not always equal to $B - A$

  $A - B \neq B . A$

- If $A \cap B = \phi$, then $A - B = \phi$

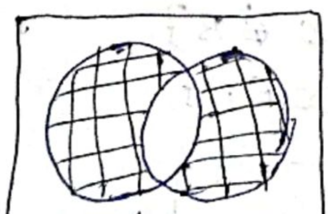
  $A - B \neq B - A$

- If $A \cap B = \phi$, then $A - B = A$, $B - A = B$

(4) Symmetric difference of set :- Let A & B are any 2 sets, then symmetric difference A & B is denoted by $A \Delta B$ & it is denoted by :

$A \Delta B = \{x : x \in A \cup B$ but $x \in A \cap B\}$

$A \Delta B = (A - B) \cup (B - A)$



$A \Delta B$

Eg:- $A = \{2, 4, 6\}$ $\quad B = \{2, 5, 8\}$

$A - B = \{4, 6\}$

$B - A = \{5, 8\}$

$A \Delta B = \{4, 6, 5, 8\}$

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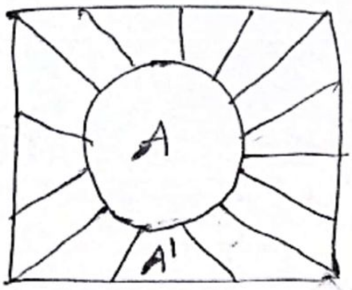
(5) **Complements of a set :-** Let 'A' be any set & 'U' is universal set, the complement of set 'A' is the set of all elements of 'U' which are not in 'A' denoted by $\bar{A}$ (or) A' (or) A^c.

i.e, $\bar{A} = \{x : x \in U \text{ and } x \notin A\}$



eg:- $U = \{1, 2, 3, 4, 5, 6\}$

$A = \{1, 3\}$

$A' = \{2, 4, 5, 6\}$

Note :-

- Usually $\bar{A} = U - A$
- $\phi' = U - \phi = U$
- $U' = U - U = \phi$
- $A \cup A' = U$
- $A \cap A' = \phi$
- $(A')' = A$

(8) Demorgan's law :

$\boxed{\overline{A \cup B} = \bar{A} \cap \bar{B} \text{ (or) } (A \cup B)' = A' \cap B'}$

$\boxed{\overline{A \cap B} = \bar{A} \cup \bar{B} \text{ (or) } (A \cap B)' = A' \cup B'}$

(9) Absorption law :

$\boxed{A \cup (A \cap B) = A}$

$\boxed{A \cap (A \cup B) = A}$

(10) Complementation law :

$\boxed{(\bar{\bar{A}}) = A}$

↳ **Laws of sets :-**

(1) Idempotent law : $\boxed{A \cup A = A}$ $\boxed{A \cap A = A}$

(2) Identity law : $\boxed{A \cup \phi = A}$ $\boxed{A \cap \phi = \phi}$

(3) Domination law : $\boxed{A \cup U = U}$ $\boxed{A \cap U = A}$ $\boxed{A \cap \phi = \phi}$

(4) (complement) Inverse law :- $\boxed{A \cup A' = U}$ $\boxed{A \cap A' = \phi}$

(5) Associative law : $\boxed{A \cup (B \cup C) = (A \cup B) \cup C}$ $\boxed{A \cap (B \cap C) = (A \cap B) \cap C}$

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(6) commutative law: $\boxed{\begin{aligned} A \cup B &= B \cup A \\ A \cap B &= B \cap A \end{aligned}}$

(7) Distributive law:

$$\boxed{\begin{aligned} A \cup (B \cap C) &= (A \cup B) \cap (A \cup C) \\ A \cap (B \cup C) &= (A \cap B) \cup (A \cap C) \end{aligned}}$$

(1) Prove that $\overline{A \cap B} = \bar{A} \cup \bar{B}$

Sol^n:- Let $x \in \overline{A \cap B}$

$x \notin A \cap B$

$x \notin A$ (or) $x \notin B$

$x \in \bar{A}$ (or) $x \in \bar{B}$

Let $x \in \bar{A} \cup \bar{B}$

$x \in \overline{A \cap B} \longleftrightarrow x \in \bar{A} \cup \bar{B}$

$\therefore \overline{A \cap B} = \bar{A} \cup \bar{B}$

(2) Prove that if $A = \{1, 3, 5, 7, 6\}$

$B = \{2, 4, 6, 1, 7\}$

$C = \{3, 7, 1, 5\}$ verify

(i) $A \cup (B \cup C) = (A \cup B) \cup C$

(ii) $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$

Sol^n (i) LHS: $A \cup (B \cup C)$

$(B \cup C) = \{1, 2, 3, 4, 5, 6, 7\}$

$A \cup (B \cup C) = \{1, 3, 5, 7, 6\} \cup \{1, 2, 3, 4, 5, 6, 7\}$

$= \{1, 2, 3, 4, 5, 6, 7\}$ — (1)

RHS: $(A \cup B) \cup C$

$(A \cup B) = \{1, 3, 5, 7, 6\} \cup \{2, 4, 6, 1, 7\}$

$= \{1, 2, 3, 4, 5, 6, 7\}$ — ~~(2)~~

$(A \cup B) \cup C = \{1, 2, 3, 4, 5, 6, 7\} \cup \{3, 7, 1, 5\}$

$= \{1, 2, 3, 4, 5, 6, 7\}$ — (2)

from (1) & (2)

$A \cup (B \cup C) = (A \cup B) \cup C$

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(ii) $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$

LHS : $A \cap (B \cup C)$

$(B \cup C) = \{2, 4, 6, 1, 7\} \cup \{3, 7, 1, 5\}$

$= \{1, 2, 3, 4, 5, 6, 7\}$

$A \cap (B \cup C) = \{1, 3, 5, 7, 6\} \cap \{1, 2, 3, 4, 5, 6, 7\}$

$= \{1, 3, 5, 6, 7\}$ — (1)

RHS : $(A \cap B) \cup (A \cap C)$

$(A \cap B) = \{1, 3, 5, 7, 6\} \cap \{2, 4, 6, 1, 7\}$

$= \{1, 6, 7\}$

$(A \cap C) = \{1, 3, 5, 7, 6\} \cap \{3, 7, 1, 5\}$

$= \{1, 3, 5, 7\}$

$(A \cap B) \cup (A \cap C) = \{1, 7, 6\} \cup \{1, 3, 5, 7\}$

$= \{1, 3, 5, 6, 7\}$ — (2)

from (1) & (2)

$A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$

(3) Prove that $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$

Soln :- Let $x \rightarrow A \cup (B \cap C)$ this implies

$x \rightarrow A$ (or) $x \rightarrow (B \cap C)$

$x \rightarrow A$ (or) $(x \rightarrow (B)$ and $x \rightarrow (C))$

$(x \rightarrow A$ or $x \rightarrow B)$ and $(x \rightarrow A$ or $x \rightarrow C)$

$x \rightarrow (A \cup B) \cap (A \cup C)$

$\therefore A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$

(4) Prove that $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$

Soln :- Let $x \rightarrow A \cap (B \cup C)$ then implies

$x \rightarrow A$ and $x \rightarrow (B \cup C)$

$x \rightarrow A$ and $x \rightarrow B$ $x \rightarrow C$

$x \rightarrow A$ and $x - B$ or $x \rightarrow A$ and $x \rightarrow C$

$x \rightarrow (A \cap B) \cup (A \cap C)$

$\therefore A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$

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⑤ Prove that : $\overline{A \cup B} = \bar{A} \cap \bar{B}$:

Solⁿ:- Let $x \to \overline{A \cup B}$

$x \notin A \cup B$

$x \notin A$ and $x \notin B$

$x \in \bar{A}$ and $x \in \bar{B}$

$x \in \bar{A} \cap \bar{B}$ $\therefore x \in \overline{A \cup B} = x \in \bar{A} \cap \bar{B}$

$\therefore \overline{A \cup B} = \bar{A} \cap \bar{B}$

⑥ $A = \{0, 1, 2, 3, 4, 7\}$ $B = \{3, 5, 4, 8, 0\}$ $C = \{0, 1, 3, 9, 8\}$

verify: (i) $A \cap (B \cap C) = (A \cap B) \cap C$

(ii) $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$

Solⁿ:- (i) LHS $= A \cap (B \cap C)$

$(B \cap C) = \{3, 0, 8\}$

$A \cap (B \cap C) = \{0, 3\}$ — ①

RHS $= (A \cap B) \cap C$

$(A \cap B) = \{0, 3, 4\}$

$(A \cap B) \cap C = \{0, 3\}$ — ②

$\therefore$ from ① & ②

$A \cap (B \cap C) = (A \cap B) \cap C$

(ii) LHS $= A \cup (B \cap C)$

$(B \cap C) = \{3, 0, 8\}$

$A \cup (B \cap C) = \{0, 1, 2, 3, 4, 7, 8\}$ — ①

RHS $= (A \cup B) \cap (A \cup C)$

$(A \cup B) = \{0, 1, 2, 3, 4, 5, 7, 8\}$

$(A \cup C) = \{0, 1, 2, 3, 4, 7, 8, 9\}$

$(A \cup B) \cap (A \cup C) = \{0, 1, 2, 3, 4, 7, 8\}$ — ②

$\therefore$ From ① & ② : $A \cup (B \cap C) = (A \cup B) \cap (A \cap C)$

⑦ If $U = \{0, 1, 2, 3, 4, 5, 6, 7, 8, 9\}$ is universal

$A = \{2, 3, 4, 8\}$ $B = \{1, 3, 4\}$ $C = \{3, 4, 5, 6\}$

(i) $\overline{A \cup B} = \bar{A} \cap \bar{B}$

$= U - A \cup B$

$= \{0, 5, 6, 7, 9\}$ — ①

$\bar{A} = U - A = \{0, 1, 5, 6, 7, 9\}$

$\bar{B} = U - B = \{0, 2, 5, 6, 7, 8, 9\}$

$\bar{A} \cap \bar{B} = \{0, 5, 6, 7, 9\}$ — ②

from ① & ② : $\overline{A \cup B} = \bar{A} \cap \bar{B}$

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(ii) $\overline{A \cap B} = \overline{A} \cup \overline{B}$

$A \cap B = \{3, 4\}$

LHS: $\overline{A \cap B} = U - A \cap B$

$= \{0, 1, 2, 5, 6, 7, 8, 9\}$ — (1)

$\overline{A} = U - A = \{0, 1, 5, 6, 7, 9\}$

$\overline{B} = U - B = \{0, 2, 5, 6, 7, 8, 9\}$

RHS: $\overline{A} \cup \overline{B} = \{0, 1, 2, 5, 6, 7, 8, 9\}$ — (2)

from (1) & (2):

$\overline{A \cap B} = \overline{A} \cup \overline{B}$

(iii) $A - B = A \cap \overline{B}$

LHS: $A - B = \{2, 8\}$ — (1)

RHS: $A \cap \overline{B}$

$\overline{B} = U - B = \{0, 2, 5, 6, 7, 8, 9\}$

$A \cap \overline{B} = \{2, 8\}$ — (2)

from eqn (1) & (2)

$A - B = A \cap \overline{B}$

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(iv) $A - (B \cup C) = (A - B) \cup (A - C)$

$(B \cup C) = \{1, 3, 4, 5, 6\}$

LHS : $A - (B \cup C) = \{2, 8\}$ — (1)

RHS : $(A - B) \cup (A - C)$

$(A - B) = \{2, 8\}$

$(A - C) = \{2, 8\}$

$(A - B) \cup (A - C) = \{2, 8\} \cup \{2, 8\}$

$= \{2, 8\}$ — (2)

from eqn (1) & (2)

$A - (B \cup C) = (A - B) \cup (A - C)$

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(8) Verify $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$ using Venn diagram;

Sol^n:-

$A \cup B = \{x : x \in A$ or $x \in B\}$
(such that)

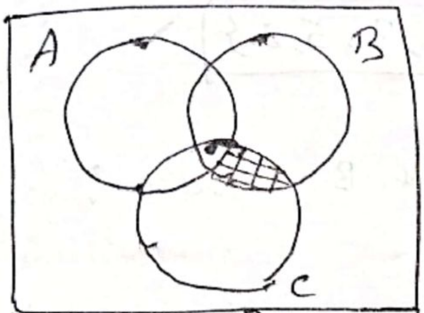
$A \cap B = \{x : x \in A$ & $x \in B\}$

$A - B = \{x : x \in A$ but $x \notin B\}$

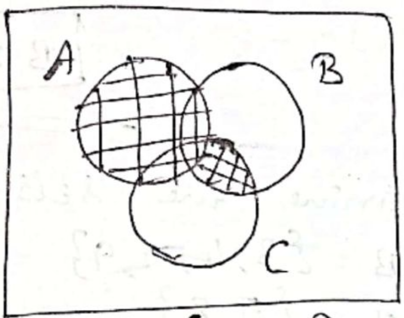
$B - A = \{x : x \in B$ but $x \notin A\}$

$A^c, \bar{A}, A' = U - A$

LHS: $A \cup (B \cap C)$

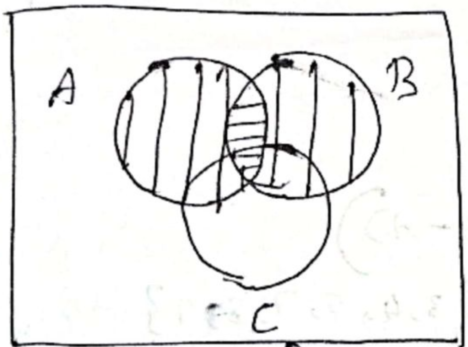
1


$(B \cap C)$

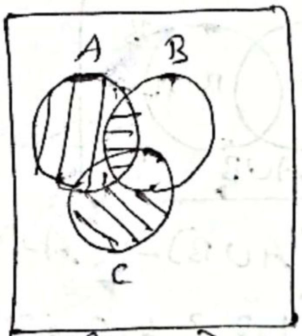
2


$A \cup (B \cap C)$

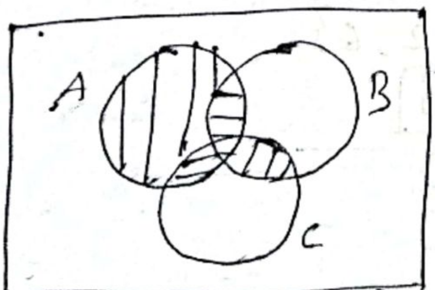
RHS: $(A \cup B) \cap (A \cup C)$

3


$(A \cup B)$

4


$(A \cup C)$

5


$(A \cup B) \cap (A \cup C)$

$\therefore$ From fig (2) & (5), it is clear that $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$

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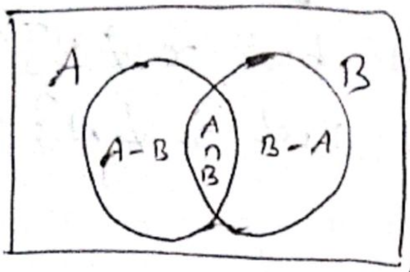
(9) Determine the sets A & B where:

$A - B = \{4, 7, 6, 9\}$

$B - A = \{3, 5, 8\}$ &

$A \cap B = \{1, 2\}$

Solⁿ:- Given: ↑



We know that:

- $A = (A - B) \cup (A \cap B)$

$= \{4, 7, 6, 9\} \cup \{1, 2\}$

$\boxed{A = \{1, 2, 4, 6, 7, 9\}}$

- $B = (A \cap B) \cup (B - A)$

$= \{1, 2\} \cup \{3, 5, 8\}$

$\boxed{B = \{1, 2, 3, 5, 8\}}$

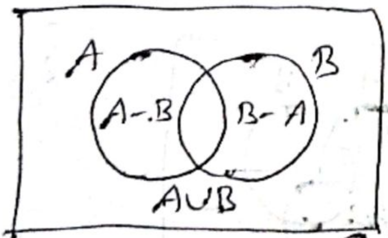
(10) Determine the sets A & B where:

$A - B = \{3, 4, 7, 9\}$

$B - A = \{5, 8\}$

$A \cup B = \{1, 2, 3, 4, 5, 6, 7, 8, 9\}$

Solⁿ:- Given: ↑



$A \cap B = (A \cup B) - ((A - B) \cup (B - A))$

$= \{1, 2, 3, 4, 5, 6, 7, 8, 9) - \{3, 4, 5, 7, 8, 9\}$

$\boxed{A \cap B = \{1, 2, 6\}}$

Now, $A = (A - B) \cup (A \cap B)$

$= \{3, 4, 7, 9\} \cup \{1, 2, 6\}$

$\boxed{A = \{1, 2, 3, 4, 6, 7, 9\}}$

$B = (A \cap B) \cup (B - A)$

$= \{1, 2, 6\} \cup \{5, 8\}$

$\boxed{B = \{1, 2, 5, 6, 8\}}$

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→ cardinality of a set :- Let 'A' be any set.

- The no of elements present in 'A' is called cardinal ~~elem~~ no of 'A' (or) cardinality of 'A'.
- It is denoted by ;

  $n(A)$ (or) $|A|$

* Applications :-

1) $n(A \cup B) = n(A) + n(B) - n(A \cap B)$
2) $n(A - B) = n(A) - n(A \cap B)$.
3) $n(B - A) = n(B) - n(A \cap B)$
4) $n(A \cup B \cup C) = n(A) + n(B) + n(C) - n(A \cap B) - n(B \cap C) - n(A \cap C) + n(A \cap B \cap C)$

① In a group of 80 people, 42 likes coffee, 60 like Tea & each person like atleast one of the two kinds. Find how many people like both coffee & tea.

Solⁿ:- Let 'A' denotes the set of people who likes coffee.

- Let 'B' denotes the set of people who like tea.

given that ; $n(A) = 42$, $n(B) = 60$, $n(A \cup B) = 80$

$n(A \cap B) = ?$

$n(A \cup B) = n(A) + n(B) - n(A \cap B)$

$80 = 42 + 60 - n(A \cap B)$

$\Rightarrow n(A \cap B) = 42 + 60 - 80$

$\Rightarrow n(A \cap B) = 22$

$\therefore$ 22 people like both coffee & tea.

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(2) In a commitee 50 people speak french 20 speaks spanish & 10 speak both french & spanish, how many speak atleast one of these 2 language.

Solⁿ:- Let 'A' be the set of people who speak french.

'B' be the set of people who speak spanish.

given that: $n(A) = 50$, $n(B) = 20$, $n(A \cap B) = 10$

$n(A \cup B) = ?$

60

$$n(A \cup B) = n(A) + n(B) - n(A \cap B)$$
$$= 50 + 20 - 10$$
$$= 50 + 10$$
$$= \underline{\underline{60}}$$

3) In a survey of 480 students in a school, 120 students listed as taking orange juice & 180 students taking apple juice & 100 students taking both orange & apple juice. Find how many students taking neither orange nor apple juice.

Solⁿ:- Let 'U' denotes the set of all students.

'A' denotes no of student who likes orange juice

'B' apple juice

given that:

$n(U) = 420$

$n(A) = 120$

$n(B) = 180$

$n(A \cap B) = 100$.

- No of students who are ready to take either apple or orange juice $= n(A \cup B)$.

$$n(A \cup B) = n(A) + n(B) - n(A \cap B)$$
$$= 120 + 180 - 100$$
$$= 200.$$

- No of students who are ~~read~~ neither take apple nor orange juice:

$n(U) = n(A \cup B)$
$= 480 - 200$
$= 280$

④ There are 300 persons with the skin disorder. 180 has been exposed to the chemical A, 75 to chemical 'B' & 45 to both A & B. Find the no of individuals exposed to:

i) chemical 'A' but not 'B',
ii) Chemical 'B' but not 'A',
iii) Chemical 'A' or chemical 'B',

Solⁿ:- Let 'U' denotes set of all persons with skin disorders.
'A' denotes set of person exposed to chemical 'A'.
'B' denotes set of person ex

given: $n(U) = 300$
$n(A) = 180$
$n(B) = 75$
$n(A \cap B) = 45$

i) No of individual exposed to 'A' but not 'B'.
$= n(A - B)$
$= n(A) - n(A \cap B)$
$= 180 - 45$
$= 135.$

ii) No of individual exposed to 'B' but not 'A'.
$= n(B - A)$
$= n(B) - n(A \cap B)$
$= 75 - 45$
$= 30,$

iii) No of individual exposed to 'A' or 'B':
$= n(A \cup B)$
$= n(A) + n(B) - n(A \cap B)$
$= 180 + 75 - 45$
$= 210$

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(5) Out of 800 boys in a school, 224 played cricket, 240 played hockey & 336 played basket ball, out of these 64 played both basket ball & hockey, 80 played both cricket & basket ball, 40 played both cricket & hockey —— 24 played all the 3 games, Find no of boys who did not played any game.

Soln:- Let 'U' denotes the set of all boys in a school

'A' denotes the set of boys who played cricket

'B' denotes the set of boys who played basket ball.

'C' denotes the set of boys who played hockey

given: $n(U) = 800$

$n(A) = 224$

$n(B) = 336$

$n(C) = 240$

$n(B \cap C) = 64$

$n(A \cap B) = 80$

$n(A \cap C) = 40$

$n(A \cap B \cap C) = 24$

- $n(A \cup B \cup C) = n(A) + n(B) + n(C) - n(A \cap B) - n(B \cap C) - n(A \cap C) + n(A \cap B \cap C)$

$= 224 + 336 + 240 - 64 - 80 - 40 + 24$

$= 640$

- $\therefore$ No of boys who did not played any game $= n(U) - A(A \cup B \cup C)$.

$= 800 - 640$

$= 160.$

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⑥ In a class consisting of 120 students, 30 has studying C++, 40 is studying Python, 45 are studying java, 15 are studying both C++ & Python & 20 are studying both Python & java, 12 are studying both C++ & java, 8 are studying all 3.

i) How many do not take any of these subjects.

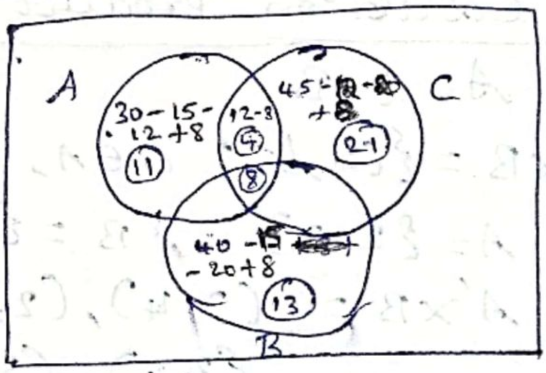
ii) How many take only one language.

Soln: i) Let 'U' denotes all students in a class.

'A' denotes the student who studying C++.

'B' denotes the student who studying Python.

'C' denotes the student who studying java.



given: $n(U) = 120$

$n(A) = 30$

$n(B) = 40$

$n(C) = 45$

$\boxed{n(A\cap B\cap C) = 8}$

$n(A\cap B) = 15$

$n(B\cap C) = 20$

$n(A\cap C) = 12$

$\cdot\ n(A\cup B\cup C) = n(A) + n(B) + n(C) - n(A\cap B) - n(B\cap C) - n(A\cap C) + n(A\cap B\cap C)$

$= 30 + 40 + 45 - 15 - 20 - 12 + 8$ $\cancel{n(A\cap B\cap C)}$

$= 76$

• No of students who do not take any of the subjects:

$= n(U) - A(A\cup B\cup C)$

$= 120 - 76$

$= 44$

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(ii) No of students studying only C++ :

$= n(A) - n(A \cap B) - n(A \cap C) + n(A \cap B \cap C)$

$= 30 - 15 - 12 + 8$

$= 11$

- No of students studying only Python :

$= n(B) - n(A \cap B) - n(B \cap C) + n(A \cap B \cap C)$

$= 40 - 15 - 20 + 8$

$= 13$

- No of students studying only java :

$= n(C) - n(A \cap C) - n(B \cap C) + n(A \cap B \cap C)$

$= 45 - 12 - 20 + 8$

$= 21$

Cartesian product of 2 sets :-

Eg:- A & B

$A \times B = \{(a, b) : a \in A, b \in B\}$

Eg:- $A = \{2, 3\}$, $B = \{4, 5, 6\}$

$A \times B = \{(2, 4), (2, 5), (2, 6), (3, 4), (3, 5), (3, 6)\}$

Eg:- $A = \phi$, $B = \{3, 4\}$

$A \times B = \phi$

- Let A & B are any two sets, then the cartesian product of A & B is denoted by $A \times B$ & is defined by :

$A \times B = \{(a, b) ; a \in A, b \in B\}$

Eg:- $A = \{2, 3\}$ $B = \{4, 5, 6\}$

$B \times A = \{(4, 2), (4, 3), (5, 2), (5, 3), (6, 2), (6, 3)\}$

$A \longrightarrow m$

$B \longrightarrow n$

$A \times B \Rightarrow mn$

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Note :-

- If set 'A' 'm' no of elements & 'B' has 'n' no of elements then $A \times B$ has $m \times n$ no of elements.

eg :- $n(A \times B) = (2)(3) = 6$

- If A = null set or B = nullset then, $A \times B$ = null set.
- $A \times B$ is always not equal to $B \times A$.

↳ Cartesian product of 3 sets :-

eg :- $A \times B \times C = \{(a,b,c) : a \in A, b \in B, c \in C\}$

$A \rightarrow m$

$B \rightarrow n$

$C \rightarrow p$

$A \times B \times C = mnp$

eg :- $A = \{1,2\}$, $B = \{3,4\}$, $C = \{5,6\}$

$A \times B \times C = \{(1,3,5), (1,3,6), (1,4,5), (1,4,6)$
$(2,3,5), (2,3,6), (2,4,5), (2,4,6)\}$

- Let A, B, C are any 3 sets, then the cartesian product of A, B, C is denoted by $A \times B \times C$ & is defined by ;

$A \times B \times C = \{(a,b,c) : a \in A, b \in B, c \in C\}$

Note :-

- If 'A' has 'm' no of elements, 'B' has 'n' no of elements and 'C' has 'p' no of elements, then $A \times B \times C$ has $m \times n \times p$ no of elements.

eg :- ~~$A = \{1,2\}$~~ $B = \{3,4\}$ $C = \{5,6\}$ $A = \{1,2\}$

$B \times C \times A = \{(3,5,1), (3,5,2), (3,6,1), (3,6,2)$
$(4,5,1), (4,5,2), (4,6,1), (4,6,2)\}$

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Eg:- $C=\{5,6\}$ $A\{1,2\}$ $B=\{3,4\}$

$C\times A\times B=\{(5,1,3),(5,1,4),(5,2,3),(5,2,4)$
$(6,1,3),(6,1,4),(6,2,3),(6,2,4)\}$

① If $G=\{2,3,7\}$ $H=\{3,6,7\}$ then find $G\times H$ & $\cancel{H}\times G$

Solⁿ:- $G\times H=\{(2,3)(2,6)(2,7)(3,3)(3,6)(3,7)$
$(7,3)(7,6)(7,7)\}$

$H\times G=\{(3,2)(3,3)(3,7)(6,2)(6,3)(6,7)$
$(7,2)(7,3)(7,7)\}$

② If $P=\{1,-1\}$ find $P\times P\times P$

Solⁿ:- $P=\{1,-1\}$ $P=\{1,-1\}$ $P=\{1,-1\}$

$P\times P\times P=\{(1,1,1)(1,1,-1)(1,-1,1)(1,-1,-1)$
$(-1,1,1)(-1,1,-1)(-1,-1,1)(-1,-1,-1)$

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(3) If $A = \{a, e, i\}$, $B = \{e, o, u\}$
$C = \{o, u, i\}$

verify (i) $A \times (B \cap C) = (A \times B) \cap (A \times C)$

(ii) $A \times (B - C) = (A \times B) - (A \times C)$

Soln:-

(i) $A \times (B \cap C) = (A \times B) \cap (A \times C)$

$$
\begin{aligned}
L.H.S &= A \times (B \cap C) \\
&= \{a, e, i\} \times \{o, u\} \\
&= \{(a, a), (a, u), (e, o), (e, u), (i, o), (i, u)\} \quad -(1)
\end{aligned}
$$

$RHS = (A \times B) \cap (A \times C)$

$$
\begin{aligned}
A \times B &= \{a, e, i\} \times \{e, o, u\} \\
&= \{(a, e), \underline{(a, o)}, \underline{(a, u)}, (e, e), \underline{(e, o)}, \\
&\quad \underline{(e, u)}, (i, e), \underline{(i, o)}, \underline{(i, u)}\}
\end{aligned}
$$

$$
\begin{aligned}
A \times C &= \{a, e, i\} \times \{o, u, i\} \\
&= \{(a, o), (a, u), (a, i), (e, o), (e, u), \\
&\quad (e, i), (i, o), (i, u), (i, i)\}
\end{aligned}
$$

$\cancel{A \times (B \cap C) = (A \times}$

$$
(A \times B) \cap (A \times C) = \{(a, o), (a, u), (e, o), (e, u), (i, o), (i, u)\} \quad -(2)
$$

from (1) & (2)

$A \times (B \cap C) = (A \times B) \cap (A \times C)$

(ii) $A \times (B - C) = (A \times B) - (A \times C)$

$LHS = A \times (B - C)$

$$
\begin{aligned}
(B - C) &= \{e, o, u\} - \{o, u, i\} \\
&= \{e\}
\end{aligned}
$$

$$
\begin{aligned}
A \times (B - C) &= \{a, e, i\} \times \{e\} \\
&= \{(a, e), (e, e), (i, e)\} \quad -(3)
\end{aligned}
$$

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$RHS = (A \times B) - (A \times C)$

$A \times B = \{a, e, i\} \times \{e, o, u\}$

$= \{(a, e), (a, o), (a, u), (e, e), (e, o), (e, u),$

$(i, e), (i, o), (i, u)\}$

$A \times C = \{a, e, i\} \times \{o, u, i\}$

$= \{(a, o), (a, u), (a, i), (e, o), (e, u), (e, i),$

$(i, o), (i, u), (i, i)\}$

$(A \times B) - (A \times C) = \{(a, e), (e, e), (i, e)\}$ — (4)

From (3) & (4)

$A \times (B - C) = (A \times B) - (A \times C)$

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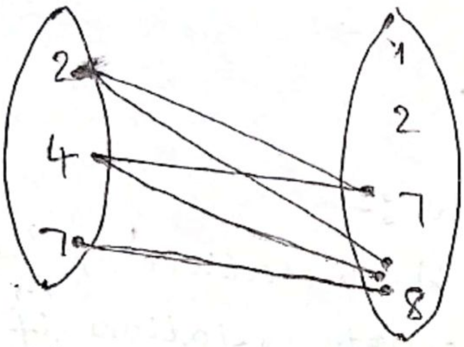
→ RELATIONS :-

eg:- $R \subseteq A \times B$

$A = \{2, 4, 7\} \quad B = \{1, 2, 7, 8\}$

$A \times B = \{(2,1), (2,2), \underline{(2,7)}, \underline{(2,8)},$
$(4,1), (4,2), \underline{(4,7)}, \underline{(4,8)},$
$(7,1), (7,2), (7,7), \underline{(7,8)}\}$ optional

$R = \{(a,b) : a \in A, b \in B, a < b\}$
$= \{(2,7), (2,8), (4,7), (4,8), (7,8)\}$



domain$(R) = \{2, 4, 7\}$
Range$(R) = \{7, 8\}$
co-domain$(R) = B$

- Let A & B are any 2 non-empty sets, then the relation R from A to B is a subset of $A \times B$
- i.e, R is a relation from A to B if & only if $\boxed{R \subseteq A \times B}$
- If added pair $(a,b) \in R$, then we say that a is related to b. & we write aRb

* Domain (R) :- The set of all 1^{st} co-ordinates of elements of 'R' is called domain of the 'R' & is denoted by : dom(R)

- i.e, $\text{dom}(R) = \{a : (a,b) \in R\}$

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* Range of relation R :- The set of all 2^{nd} coordinates of elements of 'R' is called range of R & is denoted by : Range(R).

- i.e., $Range(R) = \{b : (a,b) \in R\}$

* co-domain of the relation R :- The 2^{nd} set 'B' is called co-domain of R

Note :-

1) The 2^{nd} element is called the image of 1^{st} element.
2) Range $\subseteq$ codomain.
   (=)

↳ Types of Relations :-

① Empty relations :- A relation 'R' in a set 'A' is called empty relation if no element of 'A' is related to any element of 'A'.

i.e :- $R = \phi \subseteq A \times A$

eg :- $A = \{1, 2, 3, 5\}$ 'R' is a relation on set 'A' $A = \{1, 2, 3, 5\}$ defined by :

$R = \{(a,b) : a, b \in A \quad a + b = 0\}$

$R = \phi$

② universal relations :- A relation 'R' in a set 'A' is called universal relation if each element of 'A' is related to every element 'A'.

i.e, $R = A \times A$

eg :- $A = \{3, 5, 7\}$ $\quad A = \{3, 5, 7\}$

$R = \{(a,b) : a, b \in A, a - b < 5\}$

'R' is a relation on 'A' defined by :

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$R = \{(3,3), (3,5), (3,7), (5,3), (5,5), (5,7),$
$3-3$ $3-5$ $(7,3), (7,5), (7,7)\}$
$0<5$ -2 $-2<5$

$R = A \times A$

$\therefore$ 'R' is a universal relation.

(3) Identity relations :- The relation $I_A = \{(a,a) : \forall a \in A\}$ is called identity relation on set 'A'.

eg:- $A = \{1, 2, 5\}$

then, $I_A = \{(1,1), (2,2), (5,5)\}$

(4) Reflexive relations :- A relation 'R' in a set 'A' is said to be reflexive if every element of 'A' is related to itself.

i.e, added pair $(a,a) \in R \quad \forall a \in A$

eg:- $A = \{1, 2, 5\}$

R is a relation on set 'A' defined by :

$R = \{(a,b) : a, b \in A \quad a \leq b\}$

$= \{(1,1), (1,2), (1,5), (2,2), (2,5), (5,5)\}$

Here, $1, 2, 5 \in A \quad (1,1), (2,2), (5,5) \in R$

$\therefore$ 'R' is reflexive

Note :-

- Every identity relation is a reflexive relation, but every reflexive relations need not be a identity
- A relation 'R' on set 'A' is not reflexive if there is atleast one element $a \in A$ such that $(a,a) \notin R$.

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(5) **Symmetric relations:-** A relation 'R' in a set 'A' is said to be symmetric relation if added pair $(a,b) \in R$, then $(b,a) \in R$.

Eg:- $A = \{3, 4, 7\}$

'R' is relation on 'A' defined by:

$R = \{(3,3), (3,4), (4,3), (4,4), (7,7)\}$

∴ 'R' is symmetric

(6) **Transitive relations:-** A relation 'R' in a set 'A' is said to be transitive if added pair $(a,b), (b,c) \in R$, then $(a,c) \in R$

Eg:- $A = \{1, 2, 3, 4, 6\}$ $A = \{1, 2, 3, 4, 6\}$

"R" is relation on 'A' defined by:

$R = \{(a,b) : a \in A \; b \in A \; a \text{ divides } b\}$

$R = \{(1,1), (1,2), (1,3), (1,4), (1,6),$
$(2,2), (2,4), (2,6)$
$(3,3), (3,6)$
$(4,4), (6,6)\}$

(1,2) (2,4)
a b b c

(1,3) (3,6)
(1,4) (4,4)

2 divides 4
4 divides 8

∴ 'R' is transitive relations

(7) **Equivalance relations:-** A relation 'R' in a set 'A' is said to be equivalance relations if it is reflexive, symmetric & transitive relations,

Eg:- $A = \{1, 3, 4, 7\}$

$R = \{(1,1), (1,3), (3,1), (3,3), (4,4), (7,7),$
$(4,7), (7,4)\}$

∴ 'R' is reflexive, symmetric & transitive ∴ 'R' is equivalance relations

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① Let $A = \{1, 2, 3, 4, 5, 6\}$ define a relation 'R' from 'A' to 'A' by $R = \{(x, y) : y = x + 1, x, y \in A\}$.

i) Write elements of R.

ii) write its domain, codomain & range of R.

Solnː- i) $A = \{1, 2, 3, 4, 5, 6\}$

$A = \{1, 2, 3, 4, 5, 6\}$

$R = \{(x, y) : \underline{y = x + 1}\}$

$R = \{(1, 2), (2, 3), (3, 4), (4, 5), (5, 6)\}$

ii) • $dom(R) = \{1, 2, 3, 4, 5\}$

• $Range(R) = \{2, 3, 4, 5, 6\}$

• $codomain(R) = A$

② $A = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14\}$ define a relation 'R' on set 'A' by $R = \{(x, y) : 3x - y = 0, x, y \in A\}$

i) write elements of R.

ii) write its domain, codomain, range of R

Solnː-

i) $A = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14\}$

$A = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14\}$

$R = \{(x, y) : 3x - y = 0, x, y \in A\}$

$R = \{(1, 3), (2, 6), (3, 9), (4, 12)\}$

$3x - y = 0$

$y = 3x$

$x = 1, y = 3(1) = 3$

$x = 2, y = 3(2) = 6$

$x = 3, y = 3(3) = 9$

$x = 4, y = 3(4) = 12$

ii) • $dom(R) = \{1, 2, 3, 4\}$

• $Range(R) = \{3, 6, 9, 12\}$

• $Codomain(R) = A$

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③ Let $A = \{1, 2, 3, 4, 6\}$ R be a relation on set 'A' defined by:
$R = \{(a, b) : a, b \in A$, "b is exactly divisible by a"$\}$

i) write elements of R.

ii) write domain, co-domain & range of R.

Solnº:- i) $R = \{(a, b) : a, b \in A$, "b is exactly divisible by a"$\}$

$$R = \{(1,1), (1,2), (1,3), (1,4), (1,6), (2,2), (2,4), (2,6), (3,3), (3,6), (4,4), (6,6)\}$$

ii) $\text{domain}(R) = \{1, 2, 3, 4, 6\}$

Range ~~codomain~~ $(R) = \{1, 2, 3, 4, 6\}$

$\text{codomain}(R) = A$

④ Check whether the following relations are reflexive, symmetric & transitive:

i) Relation 'R' in the set of all integers that defined by:

$R = \{(x, y) : x - y$ is an integer$\}$

Solnº:-

$Z = \{0, \pm 1, \pm 2, \pm 3 - - -\}$ (infinite)

$R = \{(x, y) : x - y$ is an integer, $x, y \in Z\}$

- $\forall x \in Z$

$x - x = 0 \in Z$

$\Rightarrow x - x$ is an integer

$\Rightarrow (x, x) \in R$

$\therefore \forall x \in Z, (x, x) \in R$, so 'R' is reflexive

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• Let $(x, y) \in R$

$\Rightarrow x - y$ is an integer

$\Rightarrow -(y - x)$ is an integer

$\Rightarrow y - x$ is also integer

$\Rightarrow (y, x) \in R$

x	y	
$2 -$	$3 =$	-1
$3 -$	$2 =$	1

$\therefore (x, y) \in R \Rightarrow (y, x) \in R$, so 'R' is symmetric.

• Let $(x, y), (y, z) \in R$

$\Rightarrow$ (implies) $x - y$ is an integer

& $y - z$ is an integer

Now, $x - z = x - y + y - z$
$= (x - y) + (y - z)$

$\therefore$ we know that addition of two integer is again a integer.

$\Rightarrow x - z$ is also integer.

$\Rightarrow (x - z) \in R$

$\therefore (x, y), (y, z) \in R \Rightarrow (x, z) \in R$, so 'R' is transitive.

(5) Show that a relation 'R' in the set $A = \{1, 2, 3, 4, 5\}$ given by: $R = \{(a, b) : |a - b|$ is even$\}$ is an equivalence relation.

Solⁿ:-

• 'R' is reflexive:

$\forall a \in A$

$|a - a| = |0| = 0$ which is even

$\Rightarrow (a, a) \in R$

$\therefore \forall a \in A, (a, a) \in R$, so 'R' is reflexive.

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• 'R' is symmetric :

Let $(a, b) \in R$

$\Rightarrow (a-b)$ is even

$\Rightarrow |-(b-a)|$ is even

$\Rightarrow |b-a|$ is even

$\Rightarrow (b, a) \in R$

$\therefore (a,b) \in R \Rightarrow (b,a) \in R$, so 'R' is symmetric.

• 'R' is transitive :

Let $(a, b), (b, c) \in R$

$\Rightarrow |a-b|$ is even & $|b-c|$ is even

$\Rightarrow (a-b) = \underset{\text{any multiple of 2}}{2m}$ $(b-c) = 2n$

Now, $|a-c| = |(a-b) + (b-c)|$

$= |2m + 2n|$

$= |2(m+n)|$

$\Rightarrow |a-c|$ is also even

$\therefore (a, b), (b, c) \in R$

$\Rightarrow (a, c) \in R$, so 'R' is transitive

$\therefore$ 'R' is reflexive, Symmetric & transitive, hence 'R' is an equivalence relation.

(OR)

$\rightarrow R = \{(a, b) : |a-b|$ is even$\}$

$R = \{(1,1)\ (1,3)\ (1,5)\ (2,2)\ (2,4)$
$(3,1)\ (3,3)\ (3,5)\ (4,2)\ (4,4)$
$(5,1)\ (5,3)\ (5,5)\}$

- $(1,1), (2,2), (3,3), (4,4), (5,5) \in R$

$\therefore$ 'R' is reflexive

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$\rightarrow (1,3) \in R \Rightarrow (3,1) \in R$

$(1,5) \in R \Rightarrow (5,1) \in R$

$(2,4) \in R \Rightarrow (4,2) \in R$

$(3,5) \in R \Rightarrow (5,3) \in R$

$\therefore$ 'R' is symmetric

$\rightarrow (1,3),(3,1) \in R \Rightarrow (1,1) \in R$

$(1,5),(5,1) \in R \Rightarrow (1,1) \in R$

$(2,4),(4,2) \in R \Rightarrow (2,2) \in R$

$(3,5),(5,3) \in R \Rightarrow (3,3) \in R$

$(3,1),(1,3) \in R \Rightarrow (3,3) \in R$

$(5,1),(1,5) \in R \Rightarrow (5,5) \in R$

$(4,2),(2,4) \in R \Rightarrow (4,4) \in R$

$(5,3),(3,5) \in R \Rightarrow (5,5) \in R$

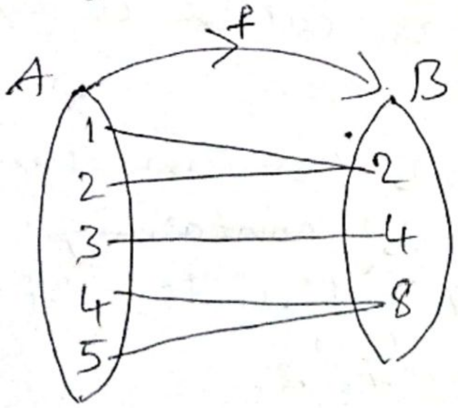
$\therefore$ 'R' is transitive

→ FUNCTIONS:-

Eg:- A & B

'f' is a relation from 'A' to 'B'

If every element of A has unique (only one relation) image in 'B'.



$f(1) = 2$

$f(2) = 2$

$f(3) = 4$

$f(4) = 8$

$f(5) = 8$

$\therefore$ It is a function

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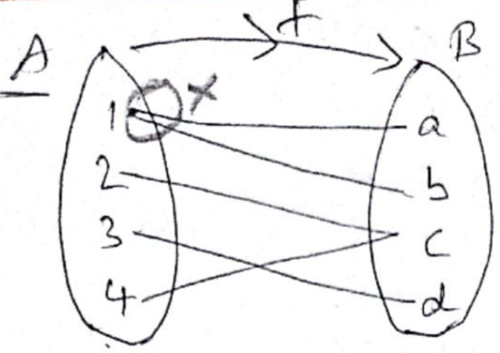
eg:-



∴ Not a function, '1' has same image in a & b

- Let A & B are 2 non-empty sets when a relation 'f' from set A to B is called function, if every element of A has unique image in B. we write;
- $f : A \rightarrow B$

Note :-

- For each A belongs to set A, there exists B belongs to set B, such that (a, b) belongs to 'f'. & we write,

  $(a, b) \in f$

  $f(a) = b$
- Here 'b' is called image of 'a' & 'a' is called preimage of 'b' under 'f'

* domain & co-domain :- $f : A \rightarrow B$, the set 'A' is called domain of function 'f' & 'B' is called co domain of function 'f'.

* Range :- if $f : A \rightarrow B$ is an function then the subset of 'B' containing images of elements of 'A' under 'f', is called Range of 'f'.

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Types of functions :-

(i) one - one function :- (Injective)

- A function $f : x \to y$ is said to be one - one function, if different element of 'x' (domain) has different images in 'y' (co-domain).

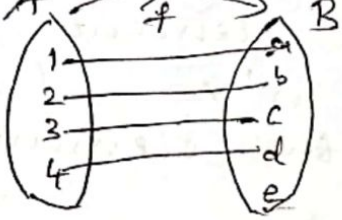
i.e, for every x_1, $x_2 \in x$ (domain)

such that if $f(x_1) = f(x_2)$

$\Rightarrow x_1 = x_2$

eg:- $A = \{1, 2, 3, 4\}$, $B = \{a, b, c, d, e\}$

$f : A \to B$ defined by :



$f(1) = a$

$f(2) = b$

$f(3) = c$

$f(4) = d$

is one - one function.

(ii) on - to (surjective) function :-

- A function $f : x \to y$ is said to be on - to, if every element of 'y' is the image of some element of 'x' under 'f'.

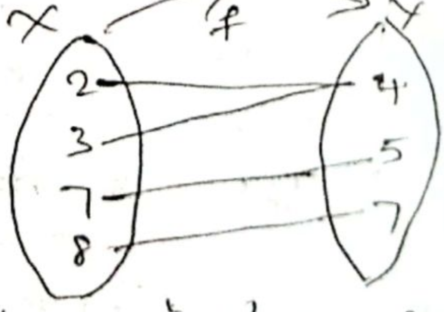
i.e, for every $y \in Y$ (codomain)

(such that) $\exists$ atleast one $x \in X$ (domain

such that $f(x) = Y$

eg:- Let $X = \{2, 3, 7, 8\}$, $Y = \{4, 5, 7\}$

A function $f : X \to Y$ defined by



$f(2) = 4$

$f(3) = 4$

$f(7) = 5$

$f(8) = 7$

is on - to function.

$\because$ all the element of 'Y' are image of some element of 'X'.

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(iii) one - one & on-to (Bijective) function:-

- A function $f: X \rightarrow Y$ is said to be bijective function, if it is both one-one & on-to function.

eg:- $f: X \rightarrow Y$

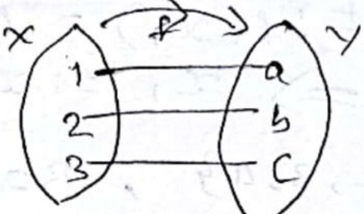
$X = \{1, 2, 3\}$ $Y = \{a, b, c\}$

A function $f: X \rightarrow Y$ defined by

$f(1) = a$

$f(2) = b$

$f(3) = c$



is bijective function.

$\therefore$ All the (different) element of 'X' has different image in 'Y'

& all the elements of 'Y' are images.

problems:-

(1) Show that the f^n $f: N \rightarrow N$ given by $f(x) = 2x$ is one - one but not on-to function.

Soln:- • 'f' is one-one :

$\forall x_1, x_2 \in \overset{R}{N}$ (domain)

such that $f(x_1) = f(x_2)$

$\cancel{2}x_1 = \cancel{2}x_2$

$\Rightarrow x_1 = x_2$

$\therefore$ 'f' is one - one function.

• 'f' is not on-to :

$\forall y \in \overset{R}{N}$ (codomain)

such that $f(x) = y$

$2x = y$

$x = \frac{y}{2} \underset{\in R}{\notin} N$ (domain)

$\therefore$ 'f' is not on-to function.

[$\because x, y \in R$ (codomain) such that $x \in R$]

(2) Check the ~~bije~~ injectivity (one-one) & surjection (on-to) of the following function;

$f : N \longrightarrow N$ defined by $f(x) = x^2$

Sol^n:- (•) if $x_1, x_2 \in N$ (domain) natural no

$f(x_1) = f(x_2)$

$x_1^{\cancel{2}} = x_2^{\cancel{2}}$

$\Rightarrow \boxed{x_1 = x_2}$

$\therefore$ 'f' is one-one function (injective)

• $x_1, x_2 \in R$

$f : R \rightarrow R$

$f(x) = x^2$

$f(x_1) = f(x_2)$

$x_1^2 = x_2^2$

$x_1 = \sqrt{x_2^2}$

$\Rightarrow x_1 = \pm x_2$

$\boxed{\begin{array}{l} x_1 = x_2 \\ x_1 = -x_2 \end{array}}$ $\therefore$

$f(x) = x^2$

$f(2) = 2^2$
$= 4$

$f(-2) = (-2)^2$
$= 4$

(•) $\forall\ y \in N$ (codomain)

such that $f(x) = y$

$x^2 = y$

$\Rightarrow x = \sqrt{y} \notin N$ (domain)

$\therefore$ 'f' is not on-to function.

• $\forall\ y \in R$ (codomain)

$\exists\ f(x) = y$

$x^2 = y$

$x = \sqrt{y}$ } not real no, it is imaginary no

$\sqrt{-3}$ }

$\sqrt{-3} \in R$ (domain)

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(3) $f : R \longrightarrow R$ defined by $f(x) = 3 - 4x$ check it is one-one or on-to function.

Soln:- if $x_1, x_2 \in N$ (domain)

$$f(x_1) = f(x_2)$$

$$3 - 4x_1 = 3 - 4x_2 \Rightarrow x_1 = x_2$$

∴ 'f' is one-one function.

$$f(x) = y$$

$$3 - 4x = y$$

$$3 - y = 4x$$

$$4x = 3 - y$$

$$x = \frac{3-y}{4}$$

∴ 'f' is on-to function.

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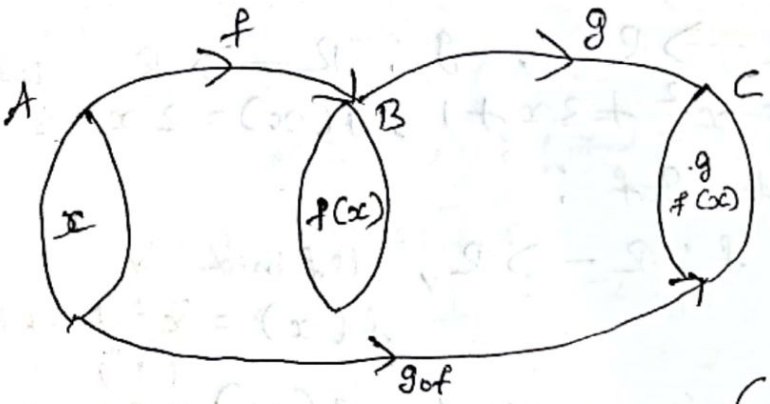
Composition of functions :- (o)

$f : A \longrightarrow B \quad , \quad g : B \longrightarrow C$

$g o f : A \longrightarrow C$



- Let 'f' from A to B $(f : A \rightarrow B)$ & 'g' from B to C $(g : B \rightarrow C)$ be two function, then composition of 'f' & 'g' denoted by gof & defined as the function: $gof : A \longrightarrow C$ given by $gof[x] = g[g(x)] \quad \forall\, x \in A$

Note :

- $fog(x) = f[g(x)]$
- $fog \neq gof$

Problems :-

① If $f : R \longrightarrow R$, $g : R \longrightarrow R$ defined by $f(x) = \sqrt{x}$, $g(x) = x^2 - 1$, then find fog & gof

Sol^n :- given $f : R \longrightarrow R$, $g : R \longrightarrow R$

$f(x) = \sqrt{x}$ ①, $\quad g(x) = x^2 - 1$ ②

- $gof(x) = g[f(x)]$
$= g[\sqrt{x}] \quad [\because \text{ by } (1)]$
$= (\sqrt{x})^2 - 1 \quad (\because \text{ by } (2))$
$= x - 1$

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- $f \circ g(x) = f[g(x)]$

$= f(x^2 - 1) \quad (\because \text{ by } (2))$

$= \sqrt{x^2 - 1}\,(x) \quad (\because \text{ by } (1))$

② If $f: R \rightarrow R$, $g: R \rightarrow R$ defined by $f(x) = x^2 + 3x + 1$, $g(x) = 2x - 3$ find $f \circ g$ & $g \circ f$:

Solⁿ:- Given, $f: R \rightarrow R$, defined by $f(x) = x^2 + 3x + 1$ (1)

$g: R \rightarrow R$, defined by $g(x) = 2x - 3$ (2)

- $g \circ f(x) = g[f(x)]$

$= g[x^2 + 3x + 1]$

$= 2(x^2 + 3x + 1) - 3$

$= 2x^2 + 6x + 2 - 3$

$= 2x^2 + 6x - 1$

- $f \circ g(x) = f[g(x)]$

$= f(2x - 3)$

$= (2x - 3)^2 + 3(2x - 3) + 1$

$= (2x)^2 + 3^2 - 12x + 6x - 9 + 1$

$= 4x^2 + \cancel{9} - 12x + 6x - \cancel{9} + 1$

$= 4x^2 - 6x + 1$

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↳ Invertible function :-

eg:- $f: x \rightarrow y$, $f(x) = y$ $\quad x \in X, y \in Y$

$f^{-1}: y \rightarrow x$, $f^{-1}(y) = x$

(or) $g(x) = y$

$f: x \rightarrow y$, there exists

$g: y \rightarrow x$, $g(y) = x$

$g \circ f(x) = x = I_x$

$f \circ g(\cancel{y}) = y = I_y$, $g(y) = f^{-1}(y)$

- Let $f: x \rightarrow y$ be one-one & on-to function & if $f(x) = y$ ~~belongs to~~ then $x \in \cancel{X}$ & $y \in Y$
$f^{-1}: y \rightarrow x$, defined by $f^{-1}(y) = x$ is called inverse of the function 'f'.

-(or)

- A function $f: x \rightarrow y$ is said to be invertible if there exist a function $g: y \rightarrow x$, such that :-

$g \circ f(x) = x = I_x$

$f \circ g(y) = y = I_y$,

then the function 'g' is called inverse of 'f' & is denoted by f^{-1}.

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(1) consider $f: R \rightarrow R$ given by $f(x) = 4x+3$ show that 'f' is invertible. find inverse of 'f'.

Sol^n:- Given,

$f: R \rightarrow R$ defined by $f(x) = 4x+3$ — (1)

- 'f' is one-one:

$\forall$ $x_1, x_2 \in R$ (domain)

such that $f(x_1) = f(x_2)$

$\Rightarrow 4x_1 + 3 = 4x_2 + 3$

$\Rightarrow 4x_1 = 4x_2 + \cancel{3} - \cancel{3}$

$\Rightarrow \cancel{4}x_1 = \cancel{4}x_2$

$\Rightarrow x_1 = x_2$

$\therefore$ 'f' is one-one function.

- 'f' is on-to:

$\forall$ $y \in R$ (codomain)

such that $f(x) = y$

$\Rightarrow 4x + 3 = y$

$\Rightarrow 4x = y - 3$

$\Rightarrow x = \frac{y-3}{4} \in R$ (domain)

$\therefore$ $\forall$ $y \in R$ (codomain)

$\exists$ atleast one $x \in R$ (domain)

$\ni$ $f(x) = y$

$\therefore$ 'f' is on-to function.

- 'f' is both one-one & on-to:

$\therefore$ 'f' is invertible.

- Inverse of f:

$f: R \rightarrow R$ defined by $f(x) = y$, $x \in R$, $y \in R$

& $f^{-1}: R \rightarrow R$ defined by $f^{-1}(y) = x$

$\Rightarrow \boxed{f^{-1}(y) = \frac{y-3}{4}}$

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[Alternative method using composition of function]

soln :- Given : $f : R \rightarrow R$ defined by :

$$f(x) = 4x + 3 = y \quad \text{--- (1)}$$

Define $g : R \rightarrow R$ defined by :

$$g(y) = x \quad \text{--- (2)}$$

From (1), $f(x) = y$

$$\Rightarrow 4x + 3 = y$$

$$\Rightarrow 4x = y - 3$$

$$\Rightarrow x = \frac{y-3}{4}$$

$$\therefore \boxed{g(y) = \frac{y-3}{4}} \quad (\because \text{ by } (2))$$

• $gof(x) = g[f(x)]$

$= g[4x + 3]$

$= \frac{(4x + \cancel{3}) - \cancel{3}}{4}$

$= \frac{\cancel{4}x}{\cancel{4}}$

$\Rightarrow gof(x) = x = I_x$

• $fog(y) = f[g(y)]$

$= f\left[\frac{y-3}{4}\right]$

$= 4\left[\frac{y-3}{4}\right] + 3$

$= y - \cancel{3} + \cancel{3}$

$\Rightarrow fog(y) = y = I_y$

$\therefore$ 'f' is invertible

$$f^{-1}(y) = g(y)$$

$$\Rightarrow \boxed{f^{-1}(y) = \frac{y-3}{4}}$$

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② If $f: N \rightarrow \cancel{N}$ is a function defined $f(x) = 4x+3$ where $Y = \{y \in N;$ $y = 4x+3$ for some $x \in N\}$ show that 'f' is invertible also find inverse of 'f':

Solⁿ:- $f: N \rightarrow Y$ defined by:

$$f(x) = 4x+3 \quad \text{——— (1)}$$

$$x = \{1, 2, 3 - -\}$$

$$Y = \{y \in N;\ y = 4x+3;\ x \in N\}$$

$$Y = \{7, 11, 15, 19, - -\}$$

- 'f' is one - one:

$\forall\ x_1; x_2 \in N$ (domain)

$$f(x_1) = f(x_2)$$

$$4x_1 + 3 = 4x_2 + 3$$

$$\cancel{4}x_1 = \cancel{4}x_2$$

$$\therefore x_1 = x_2$$

$\therefore$ 'f' is one – one function.

- 'f' is on - to:

$\forall\ y = Y$ (codomain)

$$f(x) = y$$

$$4x+3 = y$$

$$4x = y-3$$

$$x = \frac{y-3}{4} \in N \text{ (domain)}$$

$\therefore \forall\ y \in \cancel{Y}$ (codomain) $\exists$ atleast one $x \in N$ (domain)

$$f(x) = y$$

$\therefore$ 'f' is on - to function.

So, 'f' is invertible.

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Inverse of 'f':

Let $f: N \rightarrow Y$ defined by:

$f(x) = y \quad x \in N, y \in Y$

then, inverse of 'f' is:

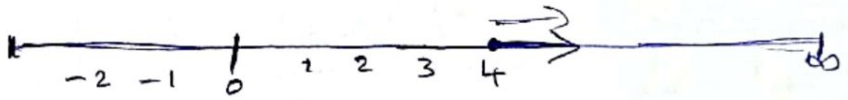
$f^{-1}: Y \rightarrow N$ defined by:

$f^{-1}(y) = x$

$$\Rightarrow \boxed{f^{-1}(y) = \frac{y-3}{4}}$$

(3) Consider $f: R_+ \rightarrow [4, \infty)$ given by $f(x) = x^2 + 4$, <u>show</u> that 'f' is invertible with inverse of 'f' is given by $f^{-1}(y) = \sqrt{y-4}$ where R_+ is the set of all non-negative real numbers:

Soln:



Given, $f: R_+ \rightarrow [4, \infty)$ given by

$f(x) = x^2 + 4$ —— (1)

- **'f' is one-one:**

$\forall x_1, x_2 \in R_+$

$f(x_1) = f(x_2)$

$x_1^2 + 4 = x_2^2 + 4$

$x_1^{\cancel{2}} = x_2^{\cancel{2}}$

$x_1 = x_2$

$\therefore$ 'f' is one-one function

- **'f' is on-to:**

$\forall y \in [4, \infty)$ (codomain)

$f(x) = y$

$x^2 + 4 = y$

$x^2 = y - 4$

$x = \sqrt{y-4} \in R_+$

$\therefore \forall y \in [4, \infty)$, atleast one $x \in R_+$

$f(x) = y$

$\therefore$ 'f' is on-to function.

So, 'f' is invertible.

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- **Inverse of 'f':**

Let $f : R+ \longrightarrow [4, \infty)$ defined by $f(x) = y$

Then inverse of 'f' is,

$f^{-1} : [4, \infty) \longrightarrow R+$ is given by

$f^{-1}(y) = x$

$$\Rightarrow \boxed{f^{-1}(y) = \sqrt{y-4}}$$

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⇒ MATHEMATICAL LOGIC :-

① Proposition (statement) :- A proposition (statement) is a sentence which is either true (or) false but not both simultaneously.

eg:- • Bangalore is in Karnataka ⟶ True

• '5' is an even number ⟶ False

② Truth-value of a statement :- If a statement is true, then its truth-value is true. & it is denoted by: T (or) 1.

• If a statement is false, then its truth-value is false & it is denoted by: F (or) 0.

• There are 2 types of statements:

(i) Simple statement :- A statement is said to be simple, if it cannot be broken down into two or more sentences

eg:- The set of real nos is an infinite set.

(ii) Compound statement :- A statement is the one which is made up of 2 or more simple-statements

Note :-

• Simple statement is denoted by : p, q, r, s.

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③ Logical connectives :- The words which combine simple statements to form compound statements are called connectives.

- In practice, we combine ~~sim~~ple (sentences) statement by using the words such as: And $(p \wedge q)$, Or $(p \vee q)$, Not $(\sim p)$, if then $(p \longrightarrow q)$, if & only if $(p \longleftrightarrow q)$.

- These words are called logical connectives,

Note:
- A statement containing 1 or more connectives is called compound statem

p	q	$p \wedge q$ (p.q)	$p \vee q$ (p+q)	$p \longrightarrow q$ (1st high then low)	$p \longleftrightarrow q$ (Both same)
0	0	0	0	1	1
0	1	0	1	1	0
1	0	0	1	0	0
1	1	1	1	1	1

↳ Fundamental of forming compound propositions :-

① Conjunction (And) :- Let p & q are 2 propositions, the compound proposition "p and q" is known as the conjunctio of p and q & it is denoted by; $p \wedge q$.

- The (Pro) conjunction of "p $\hat{\text{and}}$ q" is true if both p and q are 'true',
- In all other cases, it is 'false'.
- The truth table of the proposition: $p \wedge q$:

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p	q	p∧q
0	0	0
0	1	0
1	0	0
1	1	1

(2) Disjunction (OR) :- Let p and q are 2 propositions, the compound propositions "p or q" is called the disjunction of p and q.

- It is denoted by $p \vee q$.
- The proposition $p \vee q$ is false only one both 'p' and 'q' are false.
- In all other cases, it is true.
- The truth-table of proposition $p \vee q$:

p	q	p∨q
0	0	0
0	1	1
1	0	1
1	1	1

(3) Negation (NOT) :- Let 'p' be any proposition, the compound proposition "not p" is called negation of p.

- It is denoted by : $\sim p$.
- If a proposition 'p' is true, then the negation of 'p' is false.
- If a proposition of 'p' is false, then negation of 'p' is true.
- The truth table of negation of p:

p	~p
0	1
1	0

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(4) Conditional / Implication (if then) :-

- Let p and q are 2 propositions, the compound proposition "if p then q" is called conditional of 'p' and 'q'.
- It is denoted by : $p \longrightarrow q$.
- The proposition $p \longrightarrow q$ is false, if 'p' is true and 'q' is false.
- The truth table of $p \longrightarrow q$:

p	q	$p \longrightarrow q$
0	0	1
0	1	1
1	0	0
1	1	1

[if and only if]

(5) Double - implication (Bi-conditional) :-

- If p and q are any 2 propositions, the compound proposition "p if and only if q" is called bi-conditional proposition.
- It is denoted by : $p \longleftrightarrow q$.
- The proposition $p \longleftrightarrow q$ is true, only when p & q are true (or) both p & q are false.
- The truth table of $p \longleftrightarrow q$:

p	q	$p \longleftrightarrow q$
0	0	1
0	1	0
1	0	0
1	1	1

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Problems :-

1. Construct the truth table for the following proposition:

i) ~~Negation~~ $\sim P \wedge Q$

ii) $\sim (P \wedge Q)$

iii) $\sim P \rightarrow \sim Q$

iv) $\sim (P \vee \sim Q)$

v) $(P \vee Q) \longleftrightarrow (r \vee Q)$

2^n - values

Soln :- i)

P	Q	$\sim P$	$\sim P \wedge Q$
0	0	1	0
0	1	1	1
1	0	0	0
1	1	0	0

ii)

P	Q	$P \wedge Q$	$\sim (P \wedge Q)$
0	0	0	1
0	1	0	1
1	0	0	1
1	1	1	0

iii)

P	Q	$\sim P$	$\sim Q$	$\sim P \xrightarrow{\text{implies}} \sim Q$
0	0	1	1	1
0	1	1	0	0
1	0	0	1	1
1	1	0	0	1

iv) $\sim (P \vee \sim Q)$

P	Q	$\sim Q$	$P \vee \sim Q$	$\sim (P \vee \sim Q)$
0	0	1	1	0
0	1	0	0	1
1	0	1	1	0
1	1	0	1	0

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(1)

p	q	r	$p \vee q$	$r \vee q$	$p \vee q \longleftrightarrow r \vee q$
0	0	0	0	0	1
0	0	1	0	1	0
0	1	0	1	1	1
0	1	1	1	1	1
1	0	0	1	0	0
1	0	1	1	1	1
1	1	0	1	1	1
1	1	1	1	1	1

(2) If p, q, r are propositions with truth-value $\overset{\text{false}}{F}, \overset{\text{true}}{T}, \overset{\text{false}}{F}$ respectively, then write the truth value of:

$(p \vee r) \wedge (q \rightarrow r)$

Soln:-

p	q	r	$p \vee r$	$q \rightarrow r$	$(p \vee r) \wedge (q \rightarrow r)$
F	T	F	F	F	F

$\therefore$ The proposition:

$(p \vee r) \wedge (q \rightarrow r)$ is False.

(3) For a certain compound proposition $(p \wedge q) \longrightarrow (r \vee \underline{\sim s})$ is given to be false, find the truth-value of propositions; p, q, r & s:

Soln:- $(p \wedge q) \longrightarrow (r \vee \underline{\sim s})$ is False

$\therefore$ $p \wedge q$ is true, $r \vee \underline{\sim s}$ is false (AND)

$\therefore$ p is true, q is also true

r is false, $\sim s$ is false (OR)

$\therefore$ p is true,

q is true,

r is false,

s is true,

(or)

[Truth table]

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④ If the compound Proposition :

$\sim(r \vee \sim s) \longrightarrow \sim(p \wedge q)$

is given to be false, find the truth values of p, q, r, s :

Soln:- $\sim(r \vee \sim s) \longrightarrow \sim(p \wedge q)$ is False

$\therefore$ $\sim(r \vee \sim s)$ true, $\sim(p \wedge q)$ is false

$r \vee \sim s$ is false, $p \wedge q$ is true

$\therefore$ r is false, $\sim s$ is false, p - true

q - true

r is F, s is T, p is T, q is T

⑤ If the proposition : $(p \vee r) \wedge q$ is true, & p is false, find the truth value of q and r.

Soln:-

p	q	r	$p \vee r$	$(p \vee r) \wedge q$
0	0	0	0	0
0	0	1	1	0
0	1	0	0	0
0	1	1	1	1 ✓
1	0	0	1	0
1	0	1	1	0
1	1	0	1	1 ✓
1	1	1	1	1 ✓

p - F, q - T
r - T

p - T, q - T
r - F

p - T, q - T
r - T

$\therefore$ The proposition :

$(p \vee r) \wedge q$ is True

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→ Tautology & contradiction:-

- A compound proposition is said to be tautology, if it is always 'true' for all possible combination of truth value of its components is called <u>tautology</u>.

- A compound proposition which is always 'false' for all possible combinations of truth values of its components is called <u>contradiction</u>.

Note:

- We use the symbol T_0 to denote any tautology & F_0 to denote any contradiction

- <u>Problems:-</u>

① Show that the proposition $(p \rightarrow q) \leftrightarrow (\sim p \vee q)$ is a tautology:

<u>Soln:-</u> Given proposition: $(p \rightarrow q) \leftrightarrow (\sim p \vee q)$

p	q	$p \rightarrow q$ ①	$\sim p$	$\sim p \vee q$ ②	① $\leftrightarrow$ ②
0	0	1	1	1	1
0	1	1	1	1	1
1	0	0	0	0	1
1	1	1	0	1	1

∴ Here last column of the truth table is true & only true, hence the given proposition is a <u>tautology</u>.

② Show that the proposition $[p \rightarrow (q \rightarrow r)] \leftrightarrow [(p \rightarrow q) \rightarrow (p \rightarrow r)]$ is a tautology:

<u>Soln:-</u> Given proposition:

$$[p \rightarrow (q \rightarrow r)] \leftrightarrow [(p \rightarrow q) \rightarrow (p \rightarrow r)]$$

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p	q	r	P→r	P→(q→r) (1)	P→q	P→r	(P→q)→(P→r) (2)
0	0	0	1	1	1	1	1
0	0	1	1	1	1	1	1
0	1	0	0	1	1	1	1
0	1	1	1	1	1	1	1
1	0	0	1	1	0	0	1
1	0	1	1	1	0	1	1
1	1	0	0	0	1	0	0
1	1	1	1	1	1	1	1

(1)⟷(2)
1
1
1
1
1
1
1
1

∴ Here last column of the truth table is True & only true, Hence the given Proposition is a tautology.

(3) Show that the Proposition $(p \wedge q) \wedge \sim(p \vee q)$ is a contradiction:

Soln: Given Proposition : $(p \wedge q) \wedge \sim(p \vee q)$

(1) ∧ (2)

p	q	p∧q	p∨q	$(p\wedge q)\wedge\sim(p\vee q)$	(1)⟷(2)
0	0	0	0	0	0
0	1	0	1	0	0
1	0	0	1	0	0
1	1	1	1	0	0

$\sim(p\vee q)$
1
0
0
0

∴ Here last column of the truth-table is False & only false, hence the given Proposition is a contradiction.

Logical Equivalence :-

eg:- $S_1 \equiv S_2$

$S_1 \Longleftrightarrow S_2$

- 2 compound propositions involving same components are said to be logically equivalent if their truth values are same for each diff combinations of the truth values of their components involved in them.

i.e, If S_1 & S_2 are logically equivalent propositions, then we write:

$\boxed{S_1 \equiv S_2}$ (or) $\boxed{S_1 \Longleftrightarrow S_2}$

Laws of logic :-

(i) Idempotent laws :-

$$\boxed{\begin{aligned} p \vee p &\equiv p \\ p \wedge p &\equiv p \end{aligned}}$$

(ii) Law of double negation law :-

$$\boxed{\sim(\sim p) \equiv p}$$

(iii) Commutative laws :-

$$\boxed{\begin{aligned} p \vee q &\equiv q \vee p \\ p \wedge q &\equiv q \wedge p \end{aligned}}$$

(iv) Associative laws :-

$$\boxed{\begin{aligned} p \wedge (q \wedge r) &\equiv (p \wedge q) \wedge r \\ p \vee (q \vee r) &\equiv (p \vee q) \vee r \end{aligned}}$$

(v) Distributive laws :-

$$\boxed{\begin{aligned} p \wedge (q \vee r) &\equiv (p \wedge q) \vee (p \wedge r) \\ p \vee (q \wedge r) &\equiv (p \vee q) \wedge (p \vee r) \end{aligned}}$$

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(VI) De-Morgan's laws :-

$$\boxed{\begin{aligned} \sim(p \wedge q) &\equiv \sim p \vee \sim q \\ \sim(p \vee q) &\equiv \sim p \wedge \sim q \end{aligned}}$$

(VII) Identity laws :-

$$\boxed{\begin{aligned} p \vee F_0 &\equiv p \\ p \wedge T_0 &\equiv p \end{aligned}}$$

(VIII) Inverse laws :-

$$\boxed{\begin{aligned} p \vee \sim p &\equiv T_0 \\ p \wedge \sim p &\equiv F_0 \end{aligned}}$$

(IX) Domination laws :-

$$\boxed{\begin{aligned} p \vee T_0 &\equiv T_0 \\ p \wedge F_0 &\equiv F_0 \end{aligned}}$$

(X) Absorption laws :-

$$\boxed{\begin{aligned} p \vee (p \wedge q) &\equiv p \\ p \wedge (p \vee q) &\equiv p \end{aligned}}$$

problems :-

1) prove that $[p \rightarrow (q \wedge r)] \leftrightarrow [(p \rightarrow q) \wedge (p \rightarrow r)]$

(or) $\underbrace{[p \rightarrow (q \wedge r)]}_{1} \equiv \underbrace{[(p \rightarrow q) \wedge (p \rightarrow r)]}_{2}$

soln :-

p	q	r	$q \wedge r$	$p \rightarrow (q \wedge r)$ (1)	$(p \rightarrow q)$	$(p \rightarrow r)$	$(p \rightarrow q) \wedge (p \rightarrow r)$ (2)
0	0	0	0	1	1	1	1
0	0	1	0	1	1	1	1
0	1	0	0	1	1	1	1
0	1	1	1	1	1	1	1
1	0	0	0	0	0	0	0
1	0	1	0	0	0	1	0
1	1	0	0	0	1	0	0
1	1	1	1	1	1	1	1

Here columns (1) & (2) are identical ;

$\therefore [p \rightarrow (q \wedge r)] \equiv [(p \rightarrow q) \wedge (p \rightarrow r)]$ //

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(2) Prove that $[P \rightarrow (q \rightarrow r)] \equiv (P \wedge q) \rightarrow r$

Soln:-

P	q	r	$(q \rightarrow r)$	$(P \wedge q)$	(1) $P \rightarrow (q \rightarrow r)$	(2) $(P \wedge q) \rightarrow r$
0	0	0	1	0	1	1
0	0	1	1	0	1	1
0	1	0	0	0	1	1
0	1	1	1	0	1	1
1	0	0	1	0	1	1
1	0	1	1	0	1	1
1	1	0	0	1	0	0
1	1	1	1	1	1	1

Here (1) & (2) are identical

$\therefore [P \rightarrow (q \rightarrow r)] \equiv (P \wedge q) \rightarrow r$

(3) Show that $\sim(P \leftrightarrow q) \equiv P \leftrightarrow \sim q$

Soln:-

P	q	$P \leftrightarrow q$	(1) $\sim(P \leftrightarrow q)$	$\sim q$	(2) $P \leftrightarrow \sim q$
0	0	1	0	1	0
0	1	0	1	0	1
1	0	0	1	1	1
1	1	1	0	0	0

Here columns (1) & (2) are identical

$\therefore [\sim(P \leftrightarrow q) \equiv P \leftrightarrow \sim q]$

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(4) Prove that $\sim(P \rightarrow q) \equiv P \wedge \sim q$:

Soln :-

P	q	$(P \rightarrow q)$	$\sim(P \rightarrow q)$ (1)	$\sim q$	$P \wedge \sim q$ (2)
0	0	1	0	1	0
0	1	1	0	0	0
1	0	0	1	1	1
1	1	1	0	0	0

Here column (1) & (2) are identical

$\therefore \sim(P \rightarrow q) \equiv P \wedge \sim q$

(5) Prove that $(P \rightarrow q) \equiv [(P \wedge \sim q) \rightarrow q]$

Soln :-

P	q	$(P \rightarrow q)$ (1)	$\sim q$	$(P \wedge \sim q)$	$[(P \wedge \sim q) \rightarrow q]$ (2)
0	0	1	1	0	1
0	1	1	0	0	1
1	0	0	1	1	0
1	1	1	0	0	1

Here column (1) & (2) are identical

$\therefore (P \rightarrow q) \equiv [(P \wedge \sim q) \rightarrow q]$

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→ Converse, Inverse & contrapositive of a conditional :-

~~① converse :~~ Consider a conditional $p \to q$, then ;

- converse of $p \to q$ is : $q \to p$ (if q then p)
- Inverse of $p \to q$ is : $\sim p \to \sim q$
- contrapositive of $p \to q$ is : $\sim q \to \sim p$

① write the converse, inverse & contrapositive of the statement ;
"If x is a real no, then it is a rational no" ;

Soln:- Given statement is : "If x is a real no, then x is a rational no"

- components of the given statement are :
  p : x is a real no.
  q : x is a rational no.
- The given statement is "if p then q"
  Symbolically it is $p \to q$.
- So therefore ;
  $\sim p$: x is not a real no.
  $\sim q$: x is not a rational no.
- converse :- $p \to q$ is $q \to p$
  ∴ converse of given statement is ;
  "If x is a rational no, then it is a real no",
- Inverse :- $p \to q$ is $\sim p \to \sim q$
  ∴ Inverse of given statement is ;
  "If x is not a real no then it is not a rational no.

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- Contrapositive : $p \rightarrow q$ is $\sim q \rightarrow \sim p$.
∴ contrapositive of statement ;
"If x is not a rational no; then it is not a real no".

(2) write the converse, inverse & contrapositive for following st

i) "If two integers are equal, then their squares are equal".

ii) "If I work hard, then I get a grade".

iii) "If two triangles are equiangular, then they are similar".

iv) "If 2 is not a prime no, then 2 is an even no".

Soln :-

i) Given statement is ; "If two integers are equal, then their squares are equal".

- components of the given statement are ;
  p : two integers are equal.
  q : two integers ~~are~~ squares are equal.
- The given statement is "if p then q"
  symbollically it is $p \rightarrow q$.
- so therefore :
  $\sim p$: two integers are not equal.
- $\sim q$: two integers squares are not equal

• converse :- $p \rightarrow q$ is $q \rightarrow p$;
∴ converse of given statement is ;
" If two integers squares are equal, then two integers are equal".

• Inverse :- $p \rightarrow q$ is $\sim p \rightarrow \sim q$;
∴ Inverse of given statement is ;
" If two integers are not equal then ~~two~~ two integers squares are not equal.

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• Contrapositive :- $\sim q \rightarrow \sim p$

∴ contrapositive of given statement is :
"If two integers squares are not equal then ~~their~~ 2 integers ~~square~~ are not equal

ii) Given statement is; "If I work hard, then I get a grade",

- components :

  p : I work hard.
  q : I get a grade.

• The given statement is "if p then q"
  symbolically it is $p \rightarrow q$

∴ $\sim p$: I does not work hard.
  $\sim q$: I does not get a grade.

- converse :- $p \rightarrow q$ is $q \rightarrow p$:

∴ converse of given statement is ;
" If I get a grade, then I work hard"

- Inverse :- $p \rightarrow q$ is $\sim p \rightarrow \sim q$

∴ Inverse of given statement is :
" If I does not work hard, then I does not get a grade.

- contrapositive :- $\sim q \rightarrow \sim p$

∴ contrapositive of given statement is :
" If I does not get a grade, then I doesnot work hard.

iii) " If two triangles are equiangular, then they are similar".

- components :
  p : two triangles are equiangular.
  q : two triangles are similar.

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- The given statement is "If p then q"
Symbolically it is $p \to q$

$\therefore$ $\sim p$; two triangles are not equiangular.
$\sim q$; two triangles are not similar.

- Converse :- $p \to q$ is $q \to p$;

$\therefore$ converse of given statement is ;
"If two triangles are ~~equiangular~~ similar, then two triangles are equiangular".

- Inverse :- $p \to q$ is $\sim p \to \sim q$

$\therefore$ Inverse of given statement is ;
"If two triangles are not equiangular, then two triangles are not similar".

- Contrapositive :- $p \to q$ is $\sim q \to \sim p$

$\therefore$ Contrapositive of given statement is ;
"If two triangles are not similar, then two triangles are not equiangular.

iv) "If 2 is not a prime no, then 2 is an even no".

- Components ;
p ; 2 is not a prime no.
q ; 2 is an even no.

- The given statement is "If p then q"
Symbolically it is $p \to q$.

$\therefore$ $\sim p$; 2 is a prime no.
$\sim q$; 2 is not an even no.

Converse :- $p \to q$ is $q \to p$;

$\therefore$ converse of given statement is ;
"If 2 is an even no, then 2 is not a prime no".

Inverse :- $p \to q$ is $\sim p \to \sim q$

$\therefore$ Inverse of given statement is ;
"If 2 is a prime no, then 2 is not an even no.

contradiction :- $p \to q$ is $\sim q \to \sim p$
"If 2 is not an even no, then 2 is a prime no".

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↳ Negation of a compound proposition:-

1) $\sim(p \rightarrow q) \equiv p \wedge \sim q$
2) $\sim(p \wedge q) \equiv \sim p \vee \sim q$
3) $\sim(p \vee q) \equiv \sim p \wedge \sim q$
4) $\sim(\sim p) \equiv p$
5) $\sim(p \leftrightarrow q) \equiv p \leftrightarrow \sim q$

Problems:-

① Negate the following propositions:

"If a no is real then it is either rational or irrational".

Soln:- Given statement: If a no is real, then it is either rational or irrational.

- Components:

p : Number is real.

q : Number is rational.

r : Number is irrational.

Symbolically, the given statement is;

$$p \rightarrow (q \vee r)$$

- Negation of this statement is;

$$\sim(p \rightarrow (q \vee r))$$

$$\equiv p \wedge \sim(q \vee r)$$

$$\equiv p \wedge \sim q \wedge \sim r.$$

∴ Negation of given proposition is:

A no is real & it is neither rational nor irrational.

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② "4 is a divisor of 48 & 325 is not divisible by 4";

Sol^n:-

- Components:

  p : 4 is the divisor of 48.

  q : 325 is (not) divisible by 4.

- Given proposition in symbolically form is:

  $p \wedge \sim q$ / $p \wedge q$

- Negation of this statement is:

$\sim(p \wedge \sim q)$ | $\sim(p \wedge q)$

$\equiv \sim p \wedge \sim(\sim q)$ | $\equiv \sim p \vee \sim q$

$\equiv \sim p \vee q$

∴ Negation of given proposition is:
4 is not divisor of 48 (or) 325 is divisible by 4.

↳ Logical Inference (Rules of inference):-

- If 'P' & 'Q' are 2 arbitrary stmt, such as $P \longrightarrow Q$ is a tautology then we say 'P' logically implies 'Q'.
- This we denote it by: $P \Longrightarrow Q$

Note that (implies) $\Longrightarrow$ is not a connective (if then) nor statement.

- $P \Longrightarrow Q$ states that the statement, $P \Longrightarrow Q$ is a tautology.
- It is clear that 'Q' have the truth value 'T' whenever 'P' has the truth value 'T'.
- In general, if the implication:
  $P_1 \wedge P_2 \wedge P_3 - - \wedge P_n \longrightarrow Q$ is a tautology, then we say that the conclusion

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'Q' logically follows from the set P_1, P_2, P_3 so on to P_n of Premises & 'Q' is conclusion.

- During the process of establishing, the validity of the argument, we are not much concerned with the truth values of simple statement involved.
- This is because of the fact of we are only interested in only finding the condition follows from the premises or not irrespective of the truth values of this simple stmt involved in the premises & conclusion. Such arguments are called logical inferences.

* Rules of Inferences :-

Rules of Inferences	Name of the Rule
① $p \wedge q \rightarrow p$ $p \wedge q \rightarrow q$	Rule of conjuctive simplification
② $p \rightarrow p \vee q$ $q \rightarrow p \vee q$	Rule of disjunctive Amplification
③ $\sim p \rightarrow (p \rightarrow q)$	
④ $q \rightarrow (p \rightarrow q)$	
⑤ $p \rightarrow (p \wedge q)$ $q \rightarrow (p \wedge q)$	Rule of conjuction
⑥ $[(p \vee q) \wedge \sim p] \rightarrow q$	Rule of disjunctive syllogism
⑦ $[p \wedge (p \rightarrow q)] \rightarrow q$	Rule of detachment (Modus Ponens)
⑧ $[(p \rightarrow q) \wedge \sim q] \rightarrow \sim p$	Modus Tollens
⑨ $[(p \rightarrow q) \wedge (q \rightarrow r)] \rightarrow (p \rightarrow r)$	Law of syllogism
⑩ $[(p \wedge q) \wedge (p \rightarrow (q \rightarrow r))] \rightarrow r$	Rule of conditional Proof

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(11) $[(p \to r) \wedge (q \to r)] \to [(p \vee q) \to r]$	Rule for Proof by case
(12) $[(p \vee q) \wedge (p \to r) \wedge (q \to r)] \to r$	Dilemma

Problems:-

(1) Check the validity of Argument: $p \vee q$

$p \to r$

$q \to r$

$\therefore r$

Solⁿ:- Premises are:

$P_1 : p \vee q \quad , \quad P_2 : p \to r \quad , \quad P_3 : q \to r$

conclusion $Q : r$

$[(p \vee q) \wedge (p \to r) \wedge (q \to r)] \to r$

p	q	r	(1) $p \vee q$	(2) $p \to r$	(B) $q \to r$	(A) (1) $\wedge$ (2)	(C) (A) $\wedge$ (B)	$C \to r$
0	0	0	0	1	1	0	0	1
0	0	1	0	1	1	0	0	1
0	1	0	1	1	0	1	0	1
0	1	1	1	1	1	1	1	1
1	0	0	1	0	1	0	0	1
1	0	1	1	1	1	1	1	1
1	1	0	1	0	0	0	0	1
1	1	1	1	1	1	1	1	1

$\therefore$ Here last column of the truth-table i.e in a tautology the given argument is valid

(2) Determine the validity of the argument if cow jumps then the cat will dance. if the dog barks then the cow will jump. The dog will not bark. thus the cat will not dance :

Soln :- p : cow jumps
q : cat will dance
r : Dog barks

Premises are :
$P_1 : p \rightarrow q$
$P_2 : r \rightarrow p$
$P_3 : \sim r$

$$\begin{array}{l} p \rightarrow q \\ r \rightarrow p \\ \sim r \\ \hline \therefore \sim q \end{array}$$

conclusion $Q : \sim q$

$$[(p \rightarrow q) \wedge (r \rightarrow p) \wedge \sim r] \rightarrow \sim q$$

p	q	r	(A) $p \rightarrow q$	(B) $r \rightarrow p$	(1) A ∧ B	(2) $\sim r$	(X) 1 ∧ 2	(Y) $\sim q$	$X \rightarrow Y$
0	0	0	1	1	1	1	1	1	1
0	0	1	1	0	0	0	0	1	1
0	1	0	1	1	1	1	1	0	0
0	1	1	1	0	0	0	0	0	1
1	0	0	0	1	0	1	0	1	1
1	0	1	0	1	0	0	0	1	1
1	1	0	1	1	1	1	1	0	0
1	1	1	1	1	1	0	0	0	1

∴ Here last column of the truth-table are not completely true, therefore it is not tautology & hence given argument is not valid.

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③ Establish the validity of the argument:

$p \to q$, $q \to (r \wedge s)$, $\sim r \vee (\sim t \vee u)$, $p \wedge t$ / $\therefore u$

Solⁿ:-

$p \to q$
$q \to (r \wedge s)$
$\sim r \vee (\sim t \vee u)$
$p \wedge t$

$\therefore u$

Steps	Reason
① $p \to q$	premise
② $q \to (r \wedge s)$	premise
③ $p \to (r \wedge s)$	step ① & ②, law of syllogisms premise
④ $p \wedge t$	premise
⑤ p	step ④ & rule of conjuctive simplification
⑥ $r \wedge s$	step ⑤ & ③, rule of detachment.
⑦ r	step ⑥ & rule of conjuctive simplification
⑧ $\sim r \vee (\sim t \vee u)$	premise
⑨ $\sim(r \wedge t) \vee u$	step ⑧, Associative law & Demorgan's law
⑩ t	step ④ & rule of conjuctive simplification.
⑪ $r \wedge t$	step ⑦ & ⑩ Rule of conjuction
⑫ u	step ⑨ & ⑪, law of double negation & rule of disjunctive syllogism.

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Quantifiers :-

- The propositions contain the idea of quantity like "all", "sum", "for every", "there exist", such words are called quantifiers.
- Quantifiers are 2 types:

(i) Universal quantifiers.

(ii) Existential quantifiers.

(i). The phrases like "for all", "for each", "for any" are called universal quantifiers & denoted by symbol "$\forall$".

eg:- All triangles are right angle triangles.

eg:- All rational nos are real nos.

- The phrases like "for some", "there exists", "there exist atleast one" are called existential quantifiers, & these are denoted by the symbol "$\exists$".

eg:- Some real nos are rational nos.

eg:- There is MCA students 2 likes mathematics but not logics.

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→ MATHEMATICAL INDUCTION :-

↳ Well ordering Principle :-

- Every non-empty ~~set~~ subset of Z^+ contains a smallest element.

↳ Mathematical induction :-

Principle of mathematic induction :-

- Let 'n' be a positive integer, a proposition $s(n)$ which may be true or false, if :

1. $s(n)$ is true for $n=1$.
2. Assume that $s(K)$ is true, $K \in Z^+$
3. we should prove that $s(K+1)$ is true

Problems :-

1. Prove by mathematical induction Principle ; $1+2+3+ \cdots +n = \frac{n(n+1)}{2}$ for all the integer 'n'.

Soln :- Given $s(n) = 1+2+3+ \cdots +n = \frac{n(n+1)}{2}$

Step-1 :- If $n=1$

$$LHS = 1 \quad , \quad RHS = \frac{1(1+1)}{2} = 1$$

$$\therefore LHS = RHS$$

$\therefore$ $s(n)$ is true for $n=1$

Step-2 :- Assume that $s(n)$ is true for $n=K$

i.e, $\underset{(LHS)}{1+2+3+ \cdots +K} = \underset{(RHS)}{\frac{K(K+1)}{2}}$ — (1)

Step-3 :- we should prove that $s(n)$ is true for $n=K+1$

i.e, $1+2+3+ \cdots +(K+1) = \frac{(K+1)(K+2)}{2}$

(LHS) (RHS)

consider,

$$LHS = 1+2+3+ \cdots + (K+1)$$

$$= 1+2+3+ \cdots + \underbrace{K}_{\text{previous term}} + (K+1)$$

$$= \frac{K(K+1)}{2} + (K+1) \longrightarrow (\because \text{ by } (1))$$

$$= (K+1)\left[\frac{K}{2}+1\right]$$

$$= (K+1)\left[\frac{K+2}{2}\right] = \frac{(K+1)(K+2)}{2} = \underline{RHS}$$

$\therefore$ $s(n)$ is true for $n = K+1$

$\therefore$ $s(n)$ is true $\forall$ +ve integer 'n'.

(2) Prove by principle of mathematic induction:

$$1^2+2^2+3^2+ \cdots + n^2 = \frac{n(n+1)(2n+1)}{6}$$

for all +ve integer 'n'!

Soln:- Given $s(n) = 1^2+2^2+3^2+ \cdots + n^2 = \frac{n(n+1)(2n+1)}{6}$

Step-1:- $LHS = 1^2 = 1$, $RHS = \frac{1(1+1)(2(1)+1)}{6} = \frac{1(2)(3)}{6} = \frac{6}{6} = 1$

$LHS = RHS$

$\therefore$ $s(n)$ is true for $n=1$

Step-2:- Assume that $s(n)$ is true for $n=K$:

$$1^2+2^2+3^2+ \cdots + K^2 = \frac{K(K+1)(2K+1)}{6} \quad -(1)$$

Step-3:- we should prove that $s(n)$ is true for $n = K+1$

$$1^2+2^2+3^2+ \cdots + (K+1)^2 = \frac{(K+1)(K+2)(2K+3)}{6}$$

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$$LHS = 1^2 + 2^2 + 3^2 + \cdots + (k+1)^2$$

$$= 1^2 + 2^2 + 3^2 + \cdots + k^2 + (k+1)^2$$

$$= \frac{k(k+1)(2k+1)}{6} + (k+1)^2 \rightarrow \boxed{\text{using eqn-(1)}}$$

$$= \underset{\text{common}}{(k+1)}\left[\frac{k(2k+1)}{6} + (k+1)\right]$$

(LCM - 6)

$$= (k+1)\left[\frac{k(2k+1) + 6(k+1)}{6}\right]$$

$$= (k+1)\left[\frac{2k^2 + k + 6k + 6}{6}\right]$$

$$= (k+1)\left[\frac{2k^2 + 7k + 6}{6}\right]$$

(Factorising)

$$= (k+1)\left[\frac{2k^2 + 4k + 3k + 6}{6}\right]$$

$$= (k+1)\left[\frac{2k(k+2) + 3(k+2)}{6}\right]$$

$$= \left[\frac{(k+1)(k+2)(2k+3)}{6}\right] = \underline{RHS}$$

$\therefore$ $S(n)$ is true for $n = k+1$

$\therefore$ $S(n)$ is true $\forall$ +ve integer 'n'

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(3) Prove by principle of mathematical induction:

$$1^3 + 2^3 + 3^3 + --- + n^3 = \frac{n^2(n+1)^2}{4}$$

Soln:- Given $s(n) = 1^3 + 2^3 + 3^3 + --- + n^3 = \frac{n^2(n+1)^2}{4}$

Step-1:- $LHS = 1^3$
$= 1$

$RHS = \frac{1^2(1+1)^2}{4}$
$= \frac{1(2)^2}{4}$
$= \frac{4}{4} = 1$

LHS = RHS

$\therefore s(n)$ is true for $n=1$

Step-2:- Assume that $s(n)$ is true for $n=k$

$$1^3 + 2^3 + 3^3 + -- + k^3 = \frac{k^2(k+1)^2}{4} \quad -(1)$$

Step-3:- We should prove that $s(n)$ is true for $n=k+1$

$$1^3 + 2^3 + 3^3 + - + (k+1)^3 = \frac{(k+1)^2(k+2)^2}{4}$$

$LHS = 1^3 + 2^3 + 3^3 + -- + (k+1)^3$
$= 1^3 + 2^3 + 3^3 + -- + k^3 + (k+1)^3$
$= \frac{k^2(k+1)^2}{4} + (k+1)^3 \rightarrow (\because$ by (1))

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$$= (k+1)^2 \left[\frac{k^2}{4} + (k+1) \right]$$

$$= (k+1)^2 \left[\frac{k^2 + 4k + 4}{4} \right]$$

$$= \frac{(k+1)^2}{4} \left[k^2 + 2k + 2k + 4 \right]$$

$$= \frac{(k+1)^2}{4} \left[k(k+2) + 2(k+2) \right]$$

$$= \frac{(k+1)^2 (k+2)(k+2)}{4}$$

$$= \frac{(k+1)^2 (k+2)^2}{4} \quad // \quad \text{R.H.S}$$

$\therefore$ $S(n)$ is true for $n = k+1$

$\therefore$ $S(n)$ is true $\forall$ +ve integer 'n'.

(4) Prove that principle of mathematical induction, the sum of 1^{st} 'n' positive odd integers is n^2.

Soln:- $S(n) = 1 + 3 + 5 + \dots + (2n-1) = n^2$

$a = 1$
$b = 2$
nth term
$a_n = a + (n-1)d$
$= 1 + (n-1)2$
$= 1 + 2n - 2$
$= 2n - 1$

Step-1:- If $n = 1$

LHS $= 1$ RHS $= 1^2 = 1$

LHS = RHS

$\therefore S(n)$ is true for $n = 1$

Step-2:- Assume that $S(n)$ is true for $n = k$

$1 + 3 + 5 + \dots + (2k-1) = k^2$ —— (1)

Step-3:- We should prove that $S(n)$ is true for $n = k+1$.

$1 + 3 + 5 + \dots + (2k+1) = (k+1)^2$

$2(k+1) - 1$
$2(k+2) - 1$

$$\begin{aligned}
LHS &= 1 + 3 + 5 + \dots + (2k+1) \\
&= 1 + 3 + 5 + \dots + (2k-1) + (2k+1) \\
&= k^2 + (2k+1) \quad (\because \text{by} - (1)) \\
&= k^2 + 2k + 1 \\
&= k^2 + k + k + 1 \\
&= k(k+1) + 1(k+1) \\
&= (k+1)(k+1) \\
&= (k+1)^2 = RHS
\end{aligned}$$

$\therefore S(n)$ is true for $n = k+1$

$\therefore S(n)$ is true for all +ve integers 'n'

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(5) Prove by principle of mathematical induction for any natural no 'n', $2^n > n$:

Soln:- Given $S(n) : 2^n > n$ for all natural nos 'n'.

step-1:- If $n=1$

$2^1 > 1$

$\Rightarrow 2 > 1$ which is true.

$\therefore S(n)$ is true for $n=1$.

step-2:- Assume that $S(n)$ is true for $n=K$

i.e, $2^K > K$ — (1)

where, $K > 1$ — (2)

step-3:- we should ~~to~~ prove that $S(n)$ is true for $n=K+1$

i.e, $2^{K+1} > K+1$

$a^m \cdot a^n = a^{m+n}$
$2^K \cdot 2^1 = 2^{K+1}$

Now consider:

$2^{K+1} = 2^K \cdot 2 > K \cdot 2$ ($\because$ by -(1))

$\Rightarrow 2^{K+1} > 2K$

$\Rightarrow 2^{K+1} > (K+K) > K+1$ ($\because$ by -(2))

$\Rightarrow 2^{K+1} > K+1$

$\therefore S(n)$ is true for $n=K+1$

$\therefore S(n)$ is true for all natural nos 'n'.

(6) Prove by principle of mathematical induction for every positive integer 'n', $5^n + 2.3^{n+1} + 1$ is multiple of 8.

Soln:- Given $S(n) = 5^n + 2.3^{n+1} + 1$ is multiple of 8, $\forall$ +ve integer 'n'

Step-1:- If $n=1$,

$5^1 + 2.3^{1+1} + 1$

$= 5 + 2(9) + 1$

$= 24$

clearly $8(3) = 24$

$\therefore$ 24 is multiple of 8.

$\therefore$ S(n) is true for $n=1$.

Step-2:- Assume that S(n) is true for $n=k$

i.e, $5^k + 2.3^{k+1} + 1$ is multiple of 8.

$\Rightarrow 5^k + 2.3^{k+1} + 1 = 8m$ — (1)

$(m \in Z^+)$

Step-3:- We should prove that S(n) is true for $n=k+1$

i.e, $5^{k+1} + 2.3^{k+2} + 1$ is multiple of 8

Now consider,

$5^{k+1} + 2.3^{k+2} + 1$

$= 5^k.5 + 2.3^{k+1} 3 + 1$

$= 5.5^k + 6.3^{k+1} + 1$

$= 5.5^k + 6.3^{k+1} + 4.3^{k+1} - 4.3^{k+1} + 1 + 4 - 4$

$= 5.5^k + 10.3^{k+1} + 5 - 4.3^{k+1} - 4$

$= 5(5^k + 2.3^{k+1} + 1) - 4.3^{k+1} - 4$

$= 5(8m) - 4(3^{k+1} + 1)$ ($\because$ by (1))

We know that 3^{k+1} is an odd no.

$\therefore (3^{k+1} + 1)$ is an even no.

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$3^{k+1} + 1 = 2P$, $P \in Z^+$

$\therefore 5^{k+1} + 2 \cdot 3^{k+2} + 1 = 8(5m) - 4(2P)$

$= 8(5m) - 8P$

$= 8(5m - P)$

$\therefore 5^{k+1} + 2 \cdot 3^{k+1} + 1$ is multiple of 8

$\therefore$ $s(n)$ is true for $n = k+1$

$\therefore$ $s(n)$ is true $\forall$ +ve integers "n".

---

7) Prove by Principle of mathematical induction ; $3^{2n} - 1$ is divisible by 8 for all positive integer 'n'.

Soln :- Given $s(n) = 3^{2n} - 1$ is divisible by 8 $\forall$ +ve integer 'n'

Step-1 :- If $n=1$

LHS = 1, RHS : $3^{2n} - 1$

$= 3^{2(1)} - 1$

$= 3^2 - 1$

$= 8$

clearly 8 is divisible by 8

$\therefore$ $s(n)$ is true for $n=1$.

Step-2 :- Assume that $s(n)$ is true for $n=k$

$3^{2k} - 1 = 8m$ — (1)

$3^{2k} = 8m + 1$ — (1)

Step-3 :- we want to prove that $s(n)$ is true for $n = k+1$.

$3^{2(k+2)} - 1$ is divisible by 8

Now, consider :

$3^{2k+2} - 1$

$3^{2k} \cdot 3^2 - 1$

$(8m+1) \cdot 9 - 1$

$72m + 9 - 1$

$72m + 8$

$\therefore$ $3^{2n} - 1$ is divisible by 8.

$\therefore$ s(n) is true for $n=k+1$

$\therefore$ s(n) is true for all integers 'n'.

(8) Prove by principle of mathematical induction

$9^n - 8n - 1$ is multiple of 64 :

Soln:- $s(n) = 9^n - 8n - 1$ is multiple of 64.

Step-1:- If $n=1$

$9^1 - 8(1) - 1$

$9 - 8 - 1 = 0$

0 is multiple of 64,

$\therefore$ s(n) is true for $n=1$.

Step-2:- Assume that s(n) is true $n=k$

$9^k - 8k - 1$ is multiple of 64,

$9^k - 8k - 1 = 64m$ — (1)

Step-3:- We should prove s(n) is true for

$n = k+1$,

$9^{k+1} - 8(k+1) - 1 = 64m$

$9^{k+1} - 8k - 8 - 1 = 64m$.

$\Rightarrow 9^{k+1} - 8k - 9 = 64m$.

Now, consider,

$9^{k+1} - 8k - 9$

$9^k . 9 - 8k - 9$

$9^k . 9 - 8k + 64k - 64k - 9$

$9(9^k) - 72k - 9 + 64k$

$9(9^k - 8k - 1) + 64k$

$9(64m) + 64k$ ($\because$ eq^n)

$576m + 64k$

$64(9m + k)$

$9^{k+1} - 8k - 1$ is multiple of 64

$\therefore$ s(n) is true for $n=k+1$

$\therefore$ s(n) is true for all integer 'n'.

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(9) Prove by principle of mathematical induction
$4n < n^2 - 7 \ \forall \ n \in Z^+$ and $n \geq 6$:

Solⁿ:- $S(n) = 4n < n^2 - 7 \ \forall \ n \in Z^+ \ \& \ n \geq 6$

Step 1:- if $n = 6$

$4(6) < (6)^2 - 7$

$24 < 36 - 7$

$24 < 29$

$\therefore S(n)$ is true for $n = 6$.

Step - 2:- Assume that $S(n)$ is true for $n = k$.

$4(k) < k^2 - 7$ — (1)

Step - 3:- we should prove $S(n)$ is true for $n = k+1$.

$4(k+1) < (k+1)^2 - 7$

$\cancel{4k + 4 < k^2 + 1 - 7}$

$\cancel{4k + 4 < k^2 + 2k - 6}$

$\cancel{4k + 4 < k^2 + 2k - 6}$

$\cancel{\therefore 4k + 4 < k^2 + 2k - 6}$

$\cancel{\text{let } k = 6}$

$\cancel{4(6) + 4 < (6)^2 + 2(6) - 6}$

$\cancel{24 + 24 < 36 + 12 - 6}$

$\cancel{28 < 42}$

$S(n)$ is true for $n = k+1$

$\therefore S(n)$ is true for $\forall \ n \in Z^+$ & $n \geq 6$

$4k < k^2 - 7$

$\Rightarrow 4k + 4 < k^2 - 7 + 4$

$\Rightarrow 4(k+1) < k^2 - 7 + 2k + 1$

$\Rightarrow 4(k+1) < (k+1)^2 - 7$

$k \geq 6$

$\therefore 2k + 1 > 4$

$2 < 6$

$2 < 6 + 2$

31/3/22

UNIT - 2

COUNTING & RELATIONS

Basics of counting :-

1) Fundamental principle of addition (Sum Rule) OR
2) Fundamental principle of multiplication (Product Rule) AND

① Fundamental principle of addition (Sum rule (OR)) :-

- Suppose an event 'A' can occur in 'M' no of ways & an event 'B' can occur in 'N' ways, ~~then~~ & 2 events cannot occur simultaneously, then event 'A' or 'B' can occur in (M + N) ways.

Eg:- A college has 7 male faculty & 4 female faculties to teach discrete mathematics. A student can choose faculty in 7 + 4 i.e, 11 ways.

Eg:- In a class there are 10 girls & 20 boys, a teacher can select either a boy or girl to represent a function in 10 + 20 = 30 ways.

② Fundamental principle of multiplication (product rule (AND)) :-

- If an event 'A' can occur in 'M' different ways following which another event 'B' can occur in 'N' different ways, then the total no of occurance of event in the given order is (M × N) ways.

Eg:- Suppose a cricket ground has 3 entry gates & 4 exit gates, A person can enter & leave the ground 3 × 4 = 12 ways.

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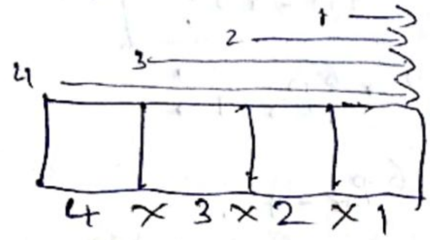
problem :-

1) Find the no of 4 letter words with or without meaning which can be formed letters of the word ~~...~~ ; LIKE , where the repetition of the letters is not allowed.

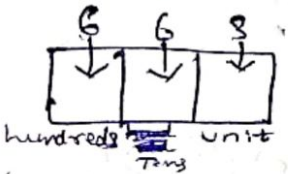
Soln :- By fundamental principle of counting required no of words is equal to $4 \times 3 \times 2 \times 1 = 24$.



Note :- If repetition of letter is allowed, then required no of word is equal to $4 \times 4 \times 4 \times 4 = 256$

2) How many 3-digit even nos can be found from the digits ; 1, 2, 3, 4, 5, 6 if the digits can be repeated ;

Soln :- 1 , 2 , 3 , 4 , 5 , 6



By fundamental principle of counting required no of 3-digit even no $= 6 \times 6 \times 3 = 108$

↳ Factorial notation :- The continued product of 1st 'n' natural nos is called factorial 'n' is denoted by $\underline{|n}$ (or) $n!$

1) $n! = n(n-1)(n-2) - - 3 \times 2 \times 1$
2) $n! = n(n-1)!$
3) $n! = n(n-1)(n-2)!$
4) $(n+1)! = (n+1)n!$
5) $0! = 1$

→ **Permutations:-** Arrangement of objects taken all at a time ~~or~~ sum at a time is called permutations.

- The no of arrangements of 'n' object taken all at a time is denoted by : ^nP_r (or) $P(n,r)$ & defined by ; $\boxed{^nP_r = \frac{n!}{(n-r)!}}$ where, $0 \le r \le n$

Problems:-

(1) Find 'r' if $^5P_r = 2 \cdot {^6P_{r-1}}$.

soln:- Given $^5P_r = 2 \cdot {^6P_{r-1}}$,

$$^nP_r = \frac{n!}{(n-r)!}$$

$$= \frac{5!}{(5-r)!} = 2\frac{6!}{(6-(r-1))!}$$

$$= \frac{5!}{(5-r)!} = \frac{2\times6\times5!}{(6-r+1)!}$$

$$= \frac{\cancel{5!}}{(5-r)!} = \frac{2\times6\times\cancel{5!}}{(7-r)!}$$

$$= \frac{1}{\cancel{(5-r)!}} = \frac{12}{(7-r)(6-r)\cancel{(5-r)!}}$$

$$\Rightarrow (7-r)(6-r) = 12$$

$$= 42 - 7r - 6r + r^2 = 12$$

$$= r^2 - 13r + 42 - 12 = 0$$

$$= r^2 - 13r + 30 = 0$$

$$= r^2 - 10r - 3r + 30 = 0$$

$$= r(r-10) - 3(r-10) = 0$$

$$= (r-10)(r-3) = 0$$

$$= r - 10 = 0 \quad \text{or} \quad r - 3 = 0$$

$$\therefore \boxed{r = 10} \quad \text{or} \quad \boxed{r = 3}$$

$r = 10$ is impossible $\therefore \boxed{r=3}$

(2) How many 4 digits nos are there with no digit repeated using the digit : 0, 1, 2, 3, 4, 5, 6, 7, 8, 9

Soln:- Given digits are ; 0, 1, 2, 3, 4, 5, 6, 7, 8, 9

No of arrangements of 10 digits taken 4 at a time is $^{10}P_4$, this includes the arrangement pair the digit '0' is at thousandth place, but such nos are 3 digit nos which are to be excluded from the total arrangements.

0 4 5 6

0			
T	H	Ten	unit

$^{10}P_4 - {}^{9}P_3$

$\therefore$ Required no of 4 digits no $= {}^{10}P_4 - {}^{9}P_3$

$$= \frac{10!}{(10-4)!} - \frac{9!}{(9-3)!}$$

$$= \frac{10 \times 9!}{6!} - \frac{9!}{6!}$$

$$= \frac{9!}{6!}[10-1] = \frac{9 \times 8 \times 7 \times \cancel{6!}}{\cancel{6!}} \times 9$$

$$= \underline{4536}$$

(3) How many words with or without meaning can be made from the letter of the word MONDAY assuming that no letter is repeated if (i)st condition : 4 letters are used at a time

(ii) All letters are used at a time.

(iii) All letters are used at a time but first letter is a vowel.

Soln:- The word MONDAY contain 6 letters

(i) This is same as no of arrangements of 6 objects taken 4 at a time,

$\therefore$ Required no of words $= {}^6P_4$

$${}^6P_4 = \frac{6!}{(6-4)!} = \frac{6 \times 5 \times 4 \times 3 \times \cancel{2!}}{\cancel{2!}} = 360$$

(ii) Required no of words $= {}^6P_6$

$${}^6P_6 = \frac{6!}{(6-6)!} = \frac{6\times5\times4\times3\times2\times1}{0} = 720$$

(iii) There are 2 vowels in given word

∴ Required no of words $= {}^2P_1 \times {}^5P_5$

$$= 2 \times 5\times4\times3\times2\times1$$
$$= 240$$

Theorem :- The no of permutations of 'n' objects taken all at a time when 'p' of them are alike of one type. 'q' of them are another type, rest are all distinct is given by :

$$\boxed{\frac{n!}{p!\,q!}}$$

ALLAHABAD $\quad \frac{9!}{4!\,2!}$ $\quad$ A - 4, L - 2

ASSAM $\quad = \frac{5!}{2!\,2!}$ $\quad$ S - 2, A - 2

Problems :-

(1) In how many ways can ~~to~~ be letters of word ALLAHABAD be arranged ?

Soln :- The word ALLAHABAD containing 9 letters out of which A is repeated 4 times & L is repeated 2 times,

∴ Required no of arrangements $= \frac{9!}{4!\,2!}$

$$= \frac{9\times8\times7\times\cancel{6}^{3}\times5\times\cancel{4!}}{\cancel{4!}\;\cancel{2}\times1}$$

$$= 7560$$

(2) In how many ways can be letters of word ASSASSINATION be arranged so that, all the ~~cases~~ S's are together ?

Solnº:- The word ASSASSINATION containing 13 letters, 'A' is repeated 3 times, 'S' is repeated 4 times, 'I' is repeated 2 times, 'N' is repeated 2 times.

∴ Total no of arrangements:

$$\frac{13!}{4!\,3!\,2!\,2!}$$

=

all

SSSS → 1 letter

ASSASSINATION

1 2 3 4 5 6 7 8 9 + 1

10

- No of arrangements where S's are together:

SSSS → Total → $\frac{4!}{4!}$ ← repeated

$$= \frac{10!}{3!\,2!\,2!} \times \frac{4!}{4!}$$

$$= \frac{10\times\cancel{9}^{3}\times\cancel{8}^{4}\times7\times\cancel{6}^{3}\times5\times4\times3\times\cancel{2}\times\cancel{1}}{\cancel{3}\times\cancel{2}\times1\times\cancel{2}\times1\times\cancel{2}\times\cancel{1}} \times \frac{\cancel{4!}}{\cancel{4!}}$$

$$= 10\times3\times4\times7\times3\times5\times4\times3$$

$$= 151200$$

3) In how many ways can be letters of word MISSISSIPPI be arranged, In how many of these arrangements do the 4 I's not come together.

Solnº:- The word MISSISSIPPI containing 11 letters, out of which 'S' is repeated 4 times, 'I' is repeated 4 times, 'P' is repeated '2' times.

∴ Total no of arrangements:

$$\frac{11!}{4!\,4!\,2!}$$

$$= \cancel{\frac{11\times10\times9\times8\times7\times6\times5\times4\times3\times2\times1}{4\times3\times2\times1\times4\times3\times2\times1\times2\times1}}$$

$$= \frac{11\times\cancel{10}^{5}\times\cancel{9}^{3}\times\cancel{8}^{2}\times7\times\cancel{6}^{3}\times5\times\cancel{4}\times\cancel{3}\times\cancel{2}\times\cancel{1}}{\cancel{4}\times\cancel{3}\times\cancel{2}\times\cancel{1}\times\cancel{4}\times\cancel{3}\times\cancel{2}\times1\times\cancel{2}\times1} = 34650$$

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(i) No of arrangements where I's are together:

MISSISSIPPI

$$= \frac{8!}{4!\,2!} \times \frac{4!}{4!}$$

$$= \frac{8 \times 7 \times 6 \times 5 \times 4 \times 3 \times 2 \times 1}{4 \times 3 \times 2 \times 1 \times 2 \times 1}$$

$= 8 \times 7 \times 3 \times 5$

$= 840$

(ii) No of arrangements where I's are not together:

= Total − I's are together

= 34650 − 840

= 33810

(4) Find the no of arrangements of letter of the word INDEPENDENCE:

In how many of these arrangements,

(i) do the words starts with P.

(ii) All the vowels always occur together.

(iii) do the vowels never occur together.

(iv) do the words begin with 'I' & end in 'P'.

Soln:- The word INDEPENDENCE containing 12 letters, out of which, 'E' is repeated 4 times 'N' is repeated 3 times and 'D' is repeated '2' times.

$\therefore$ Total $= \dfrac{12!}{4!\,3!\,2!}$

$= \dfrac{\overset{4}{\cancel{12}} \times 11 \times 10 \times \overset{4}{\cancel{8}} \times 7 \times \overset{3}{\cancel{6}} \times 5 \times \cancel{4!}}{\cancel{4!} \times \cancel{3} \times \cancel{2} \times 1 \times \cancel{2} \times 1}$

$= 4 \times 11 \times 10 \times 4 \times 7 \times 3 \times 5$

$= 184800$

(i) $\dfrac{11!}{4!\,3!\,2!}$

$= \dfrac{11 \times 10 \times \overset{3}{\cancel{9}} \times \overset{4}{\cancel{8}} \times 7 \times \overset{3}{\cancel{6}} \times 5 \times \cancel{4!}}{\cancel{4!} \times \cancel{3} \times \cancel{2} \times 1 \times \cancel{2} \times 1}$

$= 11 \times 10 \times 3 \times 4 \times 7 \times 3 \times 5$

$= 138600$ (words starts with P)

(ii) 5 - vowels (IEEEE) $\to$ 1 unit

$\dfrac{8!}{3!\,2!} \times \dfrac{5!}{4!}$

$= \dfrac{8 \times 7 \times 6 \times 5 \times \overset{2}{\cancel{4}} \times \cancel{3!}}{\cancel{3!} \times \cancel{2} \times 1} \times \dfrac{5 \times \cancel{4!}}{\cancel{4!}}$

$= \underline{8 \times 7 \times 6 \times 5 \times 2} \times \underline{5}$

$= 3360 \times 5$

$= 16800$ (vowels always occur together)

(iii) Total − vowels occur together

$184800 - 16800$

= Vowels never occur together is:

168000

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(iv).

$$\frac{10!}{4!\,3!\,2!}$$

$$= \frac{10 \times \overset{3}{\cancel{9}} \times \overset{4}{\cancel{8}} \times 7 \times \overset{3}{\cancel{6}} \times 5 \times \cancel{4!}}{\cancel{4!}\,\cancel{3} \times \cancel{2} \times 1 \times \cancel{2} \times 1}$$

$= 10 \times 3 \times 4 \times 7 \times 3 \times 5$

$=$ 12600 (words begin with 'I' and end in 'P')

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COMBINATIONS :- The no of selections of 'n' objects taken all at a time is given by ^nC_r

- The no of selections of 'n' distinct objects taken 'r' at a time is denoted by: ^nC_r (or) $C(n,r)$ & is defined by:

$$\boxed{^nC_r = \frac{n!}{r!(n-r)!}}$$ where, $0 \le r \le n$

Note :-

① $^nC_r = {}^nC_{n-r}$.

② $^nC_r + {}^nC_{r-1} = {}^{n+1}C_r$

③ $^nC_0 = \frac{n!}{0!(n-0)!} = \frac{n!}{n!} = 1$

$\therefore \boxed{^nC_0 = 1}$

④ $^nC_1 = \frac{n!}{1!(n-1)!} = \frac{n\cancel{(n-1)!}}{\cancel{(n-1)!}} = n$

$\therefore \boxed{^nC_1 = n}$

⑤ $^nC_n = \frac{\cancel{n!}}{\cancel{n!}(n-n)!}$

$= \frac{1}{0!}$

$= \frac{1}{1}$

$\boxed{^nC_n = 1}$

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Problems :-

(1) How many cards can drawn through 21 Points on a circle?

Soln:- A card can be drawn by Joining any 2 Points on a circle.

∴ The no of cards that can be drawn out of 21 points $= {}^{21}C_2$

$$= \frac{21!}{2!(21-2)!}$$

$$= \frac{21 \times \cancel{20}^{10} \times \cancel{19!}}{\cancel{2} \times 1 \times \cancel{19!}}$$

$$= 21 \times 10$$

$$= \underline{\underline{210}}$$

(2) In how many ways, can a team of 3 boys & 3 girls be selected from 5 boys & 4 girls?

Soln:- out of 5 boys, 3 boys can be selected in $5C_3$ ways.

- out of 4 girls, 3 girls can be selected in $4C_3$ ways.
- By fundamental principle, required no of selections $= 5C_3 \times 4C_3$

$$= \frac{5!}{3!(5-3)!} \times \frac{4!}{3!(4-3)!}$$

$$= \frac{5 \times \cancel{4}^{2} \times \cancel{3!}}{\cancel{3!}\,\cancel{2} \times 1} \times \frac{4 \times \cancel{3!}}{\cancel{3!} \times 1}$$

$$= 10 \times 4$$

$$= \underline{\underline{40}}$$

(3) Find the no of ways of selecting 9 - balls from 6-red balls, 5-white balls & 7-blue balls, if each selection consists of 3 - balls of each color:

Soln:- out of 6-red balls, 3- balls can be selected in $6C_3$ ways.

- out of 5-white balls, 3-balls can be selected in $5C_3$ ways.
- out of 7-blue balls, 3- balls can be selected in $7C_3$ ways.
- ∴ By fundamental principle of counting, required no of selection:

$6C_3 \times 5C_3 \times 7C_3$

$= \frac{6!}{3!(6-3)!} \times \frac{5!}{3!(5-3)!} \times \frac{7!}{3!(7-3)!}$

$= \frac{\cancel{6}^{2} \times 5 \times \cancel{4}^{2} \times \cancel{3!}}{\cancel{3 \times 2} \times \cancel{3!}} \times \frac{5 \times \cancel{4}^{2} \times \cancel{3!}}{\cancel{3!} \times \cancel{2!}} \times \frac{7 \times \cancel{6}^{3} \times 5 \times \cancel{4!}}{\cancel{3 \times 2} \times \cancel{4!}}$

$= 2 \times 5 \times 2 \times 5 \times 2 \times 7 \times 5$

$= 7000$

(4) In a examination a question paper consists 12 questions divided into parts, part-1 & 2 containing 5 & 7 questions respectively. A student is require to attempt 8 question selecting atleast 3 from each part. In how many ways can a student select the questions?

Soln:- Part-1 → 5

Part-2 → 7

12 questions

	Part-1 (5)	Part-2 (7)	(8) attempt
Possibility-1:	3	5	$[5C_3 \times 7C_5]$
2:	4	4	$[5C_4 \times 7C_4]$
3:	5	3	$[5C_5 \times 7C_3]$

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- There are 3 possibilities :

(i) 3 questions from part-1 & 5 questions from part-2

$= {}^5C_3 \times {}^7C_5$

$= \frac{5!}{3!\,(5-3)!} \times \frac{7!}{5!\,(7-5)!}$

$= \frac{5 \times 4 \times \cancel{3!}}{\cancel{3!} \times 2!} \times \frac{7 \times \overset{3}{\cancel{6}} \times \cancel{5!}}{\cancel{5!} \times \cancel{2!}}$

$= \frac{\overset{10}{\cancel{20}}}{\cancel{2}} \times 21$

$= 10 \times 21$

$= \underline{210}$

(ii) 4 questions from part-1 & 4 questions from part-2 :

$= {}^5C_4 \times {}^7C_4$

$= \frac{5!}{4!\,(5-4)!} \times \frac{7!}{4!\,(7-4)!}$

$= \frac{5!}{4! \times 1!} \times \frac{7!}{4! \times 3!}$

$= \frac{5 \times \cancel{4!}}{\cancel{4!}} \times \frac{7 \times \cancel{6} \times 5 \times \cancel{4!}}{\cancel{4!} \times 3 \times \cancel{2} \times 1}$

$= 5 \times 7 \times 5$

$= \underline{175}$

(iii) 5 questions from part-1 & 3 questions from part 2 :

$= {}^5C_5 \times {}^7C_3$

$= 1 \times \frac{7!}{3!\,4!}$

$= \underline{35}$

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∴ Required no of combinations = 210 + 175 + 35
= 420

(5) In how many ways can one select the cricket team of 11 from 17 players in which only 5 players can bowl, if each cricket team of 11 must include exactly 4 bowlers?

Soln:- Total = 17, bowlers = 5

17 − 5 = 12 → Non-bowlers

∴ $^5C_4 \times {}^{12}C_7$

∴ select 7 players who are not bowlers out of 12 players in $^{12}C_7$ ways.

• 4 bowlers can be selected out of 5 bowlers in $^5C_4$ ways

∴ Required no of ways:

$^{12}C_7 \times {}^5C_4$

$= \dfrac{12!}{7!(12-7)!} \times \dfrac{5!}{4!(5-4)!}$

$= \dfrac{12 \times 11 \times \cancel{10}^{2} \times \cancel{9}^{3} \times \cancel{8}^{2} \times \cancel{7!}}{\cancel{7!} \times \cancel{5} \times \cancel{4} \times \cancel{3} \times \cancel{2} \times 1} \times \dfrac{5 \times \cancel{4!}}{\cancel{4!} \times 1!}$

$= \cancel{6 \times 10 \times 9 \times 8 \times 5}$ $12 \times 11 \times 3 \times 2 \times 5$

$= 3,960$

at most 4 bowlers
- $^5C_4 \times {}^{12}C_7$
- $^5C_3 \times {}^{12}C_8$
- $^5C_2 \times {}^{12}C_9$
- $^5C_1 \times {}^{12}C_{10}$
- $^5C_0 \times {}^{12}C_{11}$

(+)

Atleast 4 bowlers

$^5C_5 \times {}^{12}C_6$

3,960

66 × 2
132 × 2
264 × 5
1320

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⑥ The committee of 7 has to be formed from 9 boys & 4 girls. In how many ways can this be done when the committee consists of:

i) Exactly 3 girls,
ii) atleast 3 girls,
iii) atmost 3 girls.

<u>Solnº:-</u> Total 7 has to be formed.

i) 9 - boys 4 - girls

$$\left({}^9C_4 \times {}^4C_3\right)$$

$$= \frac{9!}{4!\,(9-4)!} \times \frac{4!}{3!\,(4-3)!}$$

$$= \frac{\cancel{9}^{3} \times \cancel{8}^{2} \times 7 \times 6 \times \cancel{5} \times \cancel{4!}}{\cancel{4!} \times \cancel{5} \times \cancel{4} \times \cancel{3} \times \cancel{2} \times 1} \times \frac{4 \times \cancel{3!}}{\cancel{3!} \times 1!}$$

$$= 3 \times 7 \times 6 \times 4 = 504$$

ii) $\left({}^9C_4 \times {}^4C_3\right) + \left({}^9C_3 \times {}^4C_4\right)$

$$= \frac{9!}{3!\,(9-3)!} \times \frac{4!}{4!\,(4-4)!}$$

$$= \frac{{}^{3}\cancel{9} \times {}^{4}\cancel{8} \times 7 \times \cancel{6} \times \cancel{5} \times \cancel{4} \times \cancel{3!}}{\cancel{3!} \times \cancel{6} \times \cancel{5} \times \cancel{4} \times \cancel{3} \times \cancel{2} \times 1} \times \frac{\cancel{4!}}{\cancel{4!}}$$

$$= 3 \times 4 \times 7 = 84$$

Total: $504 + 84 = 588$

iii) $\left({}^9C_4 \times {}^4C_3\right) + \left({}^9C_5 \times {}^4C_2\right) + \left({}^9C_6 \times {}^4C_1\right) + \left({}^9C_7 \times {}^4C_0\right)$

• $\left({}^9C_4 \times {}^4C_3\right)$

$$= 504$$

• $\left({}^9C_5 \times {}^4C_2\right)$

$$\frac{9!}{5!\,(9-5)!} \times \frac{4!}{2!\,(4-2)!}$$

$$= \frac{{}^{3}\cancel{9} \times {}^{2}\cancel{8} \times 7 \times 6 \times \cancel{5!}}{\cancel{5!} \times \cancel{4} \times \cancel{3} \times \cancel{2} \times 1} \times \frac{{}^{2}\cancel{4} \times 3 \times \cancel{2!}}{\cancel{2!} \times \cancel{2} \times 1}$$

$$= 3 \times 7 \times 6 \times 2 \times 3 = 756$$

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- $\left({}^9C_6 \times {}^4C_1\right)$

$$= \frac{9!}{6!\,(9-6)!} \times \frac{4!}{1!\,(4-1)!}$$

$$= \frac{\cancel{9}^{3} \times \cancel{8}^{4} \times 7 \times \cancel{6!}}{\cancel{6!} \times \cancel{3} \times \cancel{2} \times 1} \times \frac{4 \times \cancel{3!}}{\cancel{3!}}$$

$$= 3 \times 4 \times 7 \times 4$$

$$= 336$$

- $\left({}^9C_7 \times {}^4C_0\right)$

$$= \frac{9!}{7!\,(9-7)!} \times \frac{4!}{0!\,(4-0)!}$$

$$= \frac{9 \times \cancel{8}^{4} \times \cancel{7!}}{\cancel{7!} \times \cancel{2} \times 1} \times \frac{\cancel{4!}}{\cancel{4!}}$$

$$= 9 \times 4$$

$$= 36$$

Total combinations:

$504 + 756 + 336 + 36$

$= \underline{\underline{1632}}$

→ BINOMIAL CO-EFFICIENTS :-

$(a+b)^2 = (a+b)(a+b) = a^2 + 2ab + b^2 \rightarrow$ 3 term

$(a+b)^3 = (a+b)^2(a+b) = a^3 + b^3 + 3ab(a+b)$
$= \underset{1}{a^3} + \underset{2}{b^3} + \underset{3}{3a^2b} + \underset{4}{3ab^2} \rightarrow$ 4 term

$(a+b)^4 = (a+b)^2(a+b)^2$

$(a+b)^{10} =$ Hard to calculate so, use Binomial theorem.

Binomial theorem :- Eg:-

$(a+b)^4 = {}^4C_0\, a^4 . b^0 + {}^4C_1\, a^3 . b^1 + {}^4C_2\, a^2 . b^2 + {}^4C_3\, a^1 . b^3 + {}^4C_4\, a^0 . b^4$

$= (1)a^4 + 4a^3b + 6a^2b^2 + 4ab^3 + (1)(1).b^4$

$= \underset{1}{a^4} + \underset{2}{4a^3b} + \underset{3}{6a^2b^2} + \underset{4}{4ab^3} + \underset{5}{b^4} \rightarrow$ 5 - term

$(a+b)^n \rightarrow$ 'n' is odd
(n) even → answer

- If a & b are variables & 'n' is positive integers then :

$$\boxed{(a+b)^n = {}^nC_0 . a^n . b^0 + {}^nC_1\, a^{n-1} . b^1 + {}^nC_2\, a^{n-2} . b^2 + {}^nC_3\, a^{n-3} . b^3 + \cdots + {}^nC_n . a^0\, b^n}$$

Note :-

- **General term :-** The $(r+1)^{th}$ term in the expansion $(a+b)^n$ is given by :

$\boxed{T_{r+1} = {}^nC_r . a^{n-r} . b^r}$ is called general term.

$(a+b)^4 \rightarrow n = 4$
$r = 2$

- **Middle term :-** In the expansion $(a+b)^n$, if 'n' is even then no of terms are odd, therefore only one middle term : i.e $\left[\left(\frac{n}{2}\right) + 1\right]^{th}$ term.

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- If 'n' is odd then, no of terms are even, therefore there are 2 middle term; $\left(\frac{n+1}{2}\right)^{th}$ & $\left(\frac{n+1}{2}+1\right)^{th}$ term.

Problems :-

① Write the expansion of $(2-3x)^5$:

Soln:- $(2-3x)^5 = [\underset{a}{2} + \underset{b}{(-3x)}]^5$

$= {}^5C_0 . 2^5 (-3x)^0 + {}^5C_1 . 2^4 (-3x)^1 + {}^5C_2 . 2^3 (-3x)^2 + {}^5C_3 . 2^2 (-3x)^3 + {}^5C_4 . 2^1 (-3x)^4 + {}^5C_5 (2)^0 (-3x)^5$

$= (1)(32)(1) + (5)(16)(-3)x + (10)(8)(9)x^2 + (10)(4)(-27)x^3 + (5)(2)(81)x^4 + (1)(1)(-243)x^5$

$= 32 + (-) 240x + 720x^2 + (-) 1080x^3 + 810x^4 - 243x^5$

$= 32 - 240x + 720x^2 - 1080x^3 + 810x^4 - 243x^5$

② Write co-efficient of $x^9 y^3$ in the expression $(2x-3y)^{12}$;

Soln:- Given $(2x-3y)^{12}$

Here, $n = 12$ $a = 2x$ $b = -3y$

General term is ;

$T_{r+1} = {}^nC_r . a^{n-r} . b^r$

$= {}^{12}C_r (2x)^{12-r} (-3y)^r$

$= {}^{12}C_r (2)^{12-r} x^{12-r} (-3)^r y^r$ — ①

consider, $12 - r = 9$

$\Rightarrow 12 - 9 = r$

$\Rightarrow 3 = r$

$\therefore \boxed{r = 3}$

Substitute $r = 3$ in eqn ①

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$$T_{3+1} = {}^{12}C_3 (2)^{12-3} \cdot x^{12-3} (-3)^3 \cdot y^3$$

$$= \boxed{{}^{12}C_3 (2)^9 (-3)^3} x^9 y^3$$

$\therefore$ co-efficient of $x^3 y^3$

$$= {}^{12}C_3 \, 2^9 (-3)^3$$

$$= \frac{12!}{3!\,9!} (512)(-3)^3$$

$$= \frac{\cancel{12}^{2} \times 11 \times 10 \times \cancel{9!}}{\cancel{9!} \times \cancel{3 \times 2}_{6} \times 1} (512)(-27)$$

$$= 2 \times 11 \times 10 \times 512 \times -27$$

$$= -3041280$$

③ Find co-efficient of x^7 in the expansion of $\left(x^2 + \frac{2}{x}\right)^{11}$.

Soln:- Given $\left(x^2 + \frac{2}{x}\right)^{11}$

Here, $n = 11$, $a = x^2$, $b = \left(\frac{2}{x}\right)$

General term is :

$$T_{r+1} = {}^nC_r (a)^{n-r} \cdot b^r$$

$$= {}^{11}C_r (x^2)^{11-r} \cdot \left(\frac{2}{x}\right)^r$$

$$= {}^{11}C_r \, x^{22-2r} \cdot \left(\frac{2^r}{x^r}\right)$$

$$= {}^{11}C_r \, x^{22-2r} \cdot x^{-r} \cdot 2^r$$

$$= {}^{11}C_r \cdot x^{22-2r-r} \cdot 2^r$$

$$= {}^{11}C_r \cdot 2^r \cdot x^{22-3r} \quad \text{—} ①$$

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consider, $22 - 3r = 7$

$= 22 - 7 = 3r$

$= 15 = 3r$

$= \frac{15}{3} = r$

$= 5 = r$

$= \boxed{r = 5}$

substitute $r = 5$ in (1)

$T_{5+1} = {}^{11}C_5 \cdot 2^5 \cdot x^{22-3(5)}$

$= \frac{11!}{5!\,6!}(32)x^7$

$\therefore$ co-efficient of $x^7 = \frac{11!}{5!\,6!}(32)$

$\frac{11 \times \cancel{10}^{2} \times \cancel{9}^{3} \times \cancel{8}^{2} \times 7 \times \cancel{6!}}{\cancel{5} \times \cancel{4} \times \cancel{3} \times \cancel{2} \times 1 \times \cancel{6!}}(32)$

$= 11 \times 2 \times 3 \times 7 \times 32$

$= 14784$

22×3
66×7
462×32
924
$1386+$
14784

(4) Find the term independent of 'x' in the expansion $\left(3x - \frac{1}{2x}\right)^8$

Soln:- Given $\left(3x - \frac{1}{2x}\right)^8$

Here, $n = 8$, $a = 3x$, $b = -\frac{1}{2x}$

General term is ;

$T_{r+1} = {}^nC_r (a)^{n-r} \cdot b^r$

$= {}^8C_r \cdot (3x)^{8-r} \cdot \left(\frac{-1}{2x}\right)^r$

$= {}^8C_r \cdot 3x^{8-r} \cdot \left(\frac{-1}{2x}\right)^r$

~~$= {}^8C_r \cdot 3x^{8-r} \cdot 2x^{-r} \cdot -1^r$~~

$= {}^8C_r \cdot 3^{8-r}\, x^{8-r} (-1)^r \cdot \frac{1}{(2x)^r}$

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$= {}^8C_r \cdot 3^{8-r} (-1)^r \cdot x^{8-r} \cdot \frac{1}{2^r \cdot x^r}$

$= {}^8C_r \cdot 3^{8-r} (-1)^r \cdot 2^{-r} \cdot x^{8-r} \cdot x^{-r}$

$= {}^8C_r \cdot 3^{8-r} (-1)^r \cdot 2^{-r}\, x^{8-2r}$ — (1)

Consider $8 - 2x = 0$

$\Rightarrow 8 = 2r$

$\Rightarrow \boxed{r = 4}$

Substitute $r = 4$ in (1)

$T_{4+1} = {}^8C_4\, 3^{8-4} (-1)^4 \cdot 2^{-4}\, x^{8-2(4)}$

$= \frac{8!}{4!\,4!} (3)^4 (-1)^4 \frac{1}{2^4} x^0$

$T_5 = \frac{\cancel{8} \times 7 \times \cancel{6} \times 5 \times \cancel{4!}}{\cancel{4} \times \cancel{3} \times \cancel{2} \times 1 \times \cancel{4!}} \frac{(81)(1)(1)}{\cancel{16}\, 8}$

$= \frac{7 \times 5 \times 81}{8}$

$= \frac{2835}{8}$

(5) Find the middle terms in the expansion $(2x + 3y)^9$

Soln:- Given $(2x + 3y)^9$

$a = 2x$, $b = 3y$, $n = 9$

$n = 9$ (odd) $\therefore$ no of terms in the expression is even,

$\therefore \left(\frac{n+1}{2}\right)^{th}$ term & $\left(\frac{n+1}{2} + 1\right)^{th}$ term are middle term.

$\left(\frac{9+1}{2}\right)^{th}$ & $\left(\frac{10+1}{2} + 1\right)^{th}$

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$\Rightarrow$ 5^{th} & 6^{th} terms are middle term.

General term:

$$T_{r+1} = {}^{n}C_r \cdot a^{n-r} \cdot b^r$$

$$= {}^{9}C_r \cdot (2x)^{9-r} \cdot (3y)^r \quad — (1)$$

put $\boxed{r=4}$ in (1)

$$T_{4+1} = {}^{9}C_4 \cdot (2x)^{9-4} (3y)^4$$

$$T_5 = {}^{9}C_4 \cdot 2^5 x^5 3^4 \cdot y^4$$

$$= {}^{9}C_4 \cdot 2^5 3^4 x^5 y^4$$

put $\boxed{r=5}$ in (1)

$$T_{5+1} = {}^{9}C_5 (2x)^{9-5} (3y)^5$$

$$T_6 = {}^{9}C_5 \cancel{2} 2^4 x^4 3^5 \cdot y^5$$

$$T_6 = {}^{9}C_5 2^4 3^5 x^4 y^5$$

$\therefore$ T_5 & T_6 are middle term.

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→ **Recursive definitions :-** We use 2 steps to define function with the set of non-negative integer as its ~~set~~ ~~of~~ domain :

1) **Basic steps :-** Specify the value of function at '0'.

2) **Recursive step :-** Give a rule for finding its value at an integer from its values at smaller integer, such definitions are called recursive definitions.

- The sequence of real nos can be represented in 2 ways :

i) **Explicit method :-** A sequence is represented by means of its general - term, i.e., in terms of 'n', is called explicit method.

ii) **Recursive method :-** In this method few terms of the sequence from the beginning are explicitly written & the general term is specified through a formula which containing previous elements,

Problems :- (method)

① Find recursive definitions for the following sequence ;

i) $a_n = 3^n$ ii) $a_n = 3n+7$.

Soln :-

i) $a_n = 3^n$

Put $n=1,\ a_1 = 3$

$n=2,\ a_2 = 3^2 = 9$

$n=3,\ a_3 = 3^3 = 27$

$n=4,\ a_4 = 3^4 = 81$

- 3, 9, 27, 81, — — —
- 3, 3×3, 3×3×3, 3×3×3×3. — —
- 3, $3 \cdot 3^1$, $3 \cdot 3^2$, $3 \cdot 3^3$, —

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$, \frac{3}{3}, \frac{3\times 3}{9}, \frac{9\times 3}{27}, \frac{29\times 3}{81}, ---$

$a_n = 3 \cdot a_{n-1}$ where $a_1 = 3$

$n=2, \quad a_2 = 3 \cdot a_1 = 3(3) = 9$

$n=3, \quad a_3 = 3 \cdot a_2 = 3(9) = 27$

$n=4, \quad a_4 = 3 \cdot a_3 = 3(27) = 81$

$3, 9, 27, 81, ---$

$\therefore$ Required recursive definition is:

$\boxed{a_n = 3 \cdot a_{n-1}}$ where $\boxed{a_1 = 3}$

ii) $a_n = 3n+7$

put $n=1, \quad a_1 = 3(1)+7 = a_1 = 3+7 = 10$

$n=2, \quad a_2 = 3(2)+7 = a_2 = 6+7 = 13$

$n=3, \quad a_3 = 3(3)+7 = a_3 = 9+7 = 16$

$n=4, \quad a_4 = 3(4)+7 = a_4 = 12+7 = 19$

$10, 13, 16, 19, ---$

$\therefore$ Recursive definition for this sequence is:

$\boxed{a_n = a_{n-1} + 3, \quad a_1 = 10}$

② A sequence (S_n) of integers is defined by $S_1 = 2$ & $S_n = S_{n-1} + 3$ for $n \geq 2$ find S_n explicitly:

Soln:- Given $S_n = S_{n-1} + 3$ for $n \geq 2$, $S_1 = 2$

$S_n = S_{n-1} + 3 = S_{n-1} + 1 \cdot 3$

$S_n = S_{n-2} + 3 + 3 = S_{n-2} + 2 \cdot 3$

$S_n = S_{n-3} + 3 + 6 = S_{n-3} + 3 \cdot 3$

$S_n = S_{n-4} + 3 + 9 = S_{n-4} + 4 \cdot 3$

$S_n = S_{n-1} + 3$

$S_{n-1} = S_{n-2} + 3$

$S_{n-2} = S_{n-3} + 3$

$\vdots$

$S_n = S_{n-(n-1)} + (n-1)3$

$= S_{\cancel{n}-\cancel{n}+1} + (n-1)3$

$= S_1 + (n-1)3$

$= 2 + (n-1)3$

$= 2 + 3n - 3$

$= 3n - 1$

$\boxed{S_n = 3n - 1}$

↳ ~~Recurrance relations:-~~

Fibonacci nos :- The no defined recursively $f_0 = 0$, $f_1 = 1$ & $f_n = f_{n-1} + f_{n-2}$ for $n \in Z^+$, $n \geq 2$ are called fibonacci nos,

eg:- 0, 1, 1, 2, 3, 5, 8, — —

↳ **Recurrance relations :-** A recurrance relation for the sequence $\langle a_n \rangle$ is an equation that express $\langle a_n \rangle$ in terms of 1 or more of the previous terms of the sequence.

- A sequence is called solution of the recurrance relations, if its terms satisfied the recurrance relations.

*** Solution of linear recurrance relations:-**

Theorem-1:- Let c_1 & c_2 be real nos suppose that $r^2 - c_1 r - c_2 = 0$ has two distinct roots r_1 & r_2 then the sequence $\langle a_n \rangle$ is a solution of the recurrance relation,

$a_n = c_1 a_{n-1} + c_2 a_{n-2}$ if & only if

$a_n = \alpha_1 r_1^n + \alpha_2 r_2^n$ for $n = 0, 1, 2, \ldots$

where α_1 & α_2 are constants.

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Theorem-2:- c_1 & c_2 be real nos suppose that $r^2 - c_1 r - c_2 = 0$ has one & only one root $r = r_0$, then the sequence (a_n) is a solution of the recurrance relation;

$a_n = c_1 a_{n-1} + c_2 a_{n-2}$ if & only if

$a_n = \alpha_1 r_0^n + n \alpha_2 r_0^n$ for $n = 0, 1, 2, --$

where α_1 & α_2 are constants.

Problem:-

(1) Find the solution of recurrance relation:

$a_n = a_{n-1} + 2a_{n-2}$ where $a_0 = 2$

$a_1 = 7$;

soln:- Given $a_n = a_{n-1} + 2a_{n-2}$, $a_0 = 2$

$a_1 = 7$ — (1)

characteristic equation of (1) is;

$r^2 - r - 2 = 0$

$r^2 - 2r + r - 2 = 0$

$r(r-2) + 1(r-2) = 0$

$(r-2)(r+1) = 0$

$\Rightarrow r - 2 = 0$, $r + 1 = 0$

$\Rightarrow r = 2$, $r = -1$

roots are distinct

$\therefore \boxed{r_1 = 2 \quad r_2 = -1}$

Solution of (1) is given by:

$a_n = \alpha_1 r_1^n + \alpha_2 r_2^n$

$\Rightarrow a_n = \alpha_1 (2)^n + \alpha_2 (-1)^n$ — (2)

Given $a_0 = 2$, $a_1 = 7$

Put $n = 0$ in (2)

$a_0 = \alpha_1 (2)^0 + \alpha_2 (-1)^0$

$2 = \alpha_1 (1) + \alpha_2 (1)$

$\Rightarrow \alpha_1 + \alpha_2 = 2$ — (3)

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Put $n=1$ in (2)

$a_1 = \alpha_1 (2)^1 + \alpha_2 (-1)^1$

$7 = 2\alpha_1 + \alpha_2 (-1)$ — (4)

$\Rightarrow 2\alpha_1 - \alpha_2 = 7$

adding (3) & (4)

$\alpha_1 + \cancel{\alpha_2} = 2$

$2\alpha_1 - \cancel{\alpha_2} = 7$

$3\alpha_1 = 9$

$\boxed{\alpha_1 = 3}$

Substitute $\alpha_1 = 3$ in (3)

$3 + \alpha_2 = 2$

$\Rightarrow \alpha_2 = 2 - 3$

$\boxed{\alpha_2 = -1}$

substitute $\alpha_1 = 3$, $\alpha_2 = -1$. in (2)

$a_n = 3(2)^n + (-1)(-1)^n$

$\boxed{a_n = 3(2)^n - (-1)^n}$

(2) Find the explicit formula for fibonacci nos defined by : $f_n = f_{n-1} + f_{n-2}$ where $f_0 = 0$, $f_1 = 1$

Soln:- Given

$f_n = f_{n-1} + f_{n-2}$ where $f_0 = 0$, $f_1 = 1$ — (1)

characteristic eqn of (1)

$r^2 - r - 1 = 0$ $\longrightarrow$ (cannot factorize)

$a = 1$, $b = -1$, $c = -1$

$r = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$= \dfrac{-(-1) \pm \sqrt{1 - 4(1)(-1)}}{2(1)}$

$= \dfrac{1 \pm \sqrt{5}}{2}$ $\therefore$ roots are distinct

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$$\boxed{r_1 = \frac{1+\sqrt{5}}{2}} \quad \boxed{r_2 = \frac{1-\sqrt{5}}{2}}$$

- solution of (1) is given by:

$f_n = \alpha_1 r_1^n + \alpha_2 r_2^n$

$$f_n = \alpha_1\left(\frac{1+\sqrt{5}}{2}\right)^n + \alpha_2\left(\frac{1-\sqrt{5}}{2}\right)^n \quad - (2)$$

we know that $f_0 = 0$, $f_1 = 1$

- put $n=0$ in (2) we get,

$$f_0 = \alpha_1\left(\frac{1+\sqrt{5}}{2}\right)^0 + \alpha_2\left(\frac{1-\sqrt{5}}{2}\right)^0$$

$\Rightarrow 0 = \alpha_1(1) + \alpha_2(1)$

$\Rightarrow \alpha_1 + \alpha_2 = 0$

$\Rightarrow \alpha_1 = -\alpha_2 \quad - (3)$

- put $n=1$ in (2)

$$f_1 = \alpha_1\left(\frac{1+\sqrt{5}}{2}\right)^1 + \alpha_2\left(\frac{1-\sqrt{5}}{2}\right)^1$$

$$1 = -\alpha_2\left(\frac{1+\sqrt{5}}{2}\right) + \alpha_2\left(\frac{1-\sqrt{5}}{2}\right)$$

$$1 = \alpha_2\left(\left(\frac{-1-\sqrt{5}}{2}\right) + \frac{1-\sqrt{5}}{2}\right)$$

$$1 = \alpha_2\left(\frac{\cancel{-1} - \sqrt{5} + \cancel{1} - \sqrt{5}}{2}\right)$$

$$1 = \left(\frac{-\cancel{2}\sqrt{5}}{\cancel{2}}\right) \cdot \alpha_2$$

$1 = -\sqrt{5}\,\alpha_2$

$$\therefore \boxed{\alpha_2 = \frac{-1}{\sqrt{5}}}$$

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from (3)

$\alpha_1 = -\alpha_2$

$= -\left(\frac{-1}{\sqrt{5}}\right)$

$$\boxed{\alpha_1 = \frac{1}{\sqrt{5}}}$$

substitute $\alpha_1 = \frac{1}{\sqrt{5}}$

$\alpha_2 = \frac{-1}{\sqrt{5}}$ in (2)

$$f_n = \frac{1}{\sqrt{5}}\left[\frac{1+\sqrt{5}}{2}\right]^n - \frac{1}{\sqrt{5}}\left[\frac{1-\sqrt{5}}{2}\right]^n$$

which is the recursive explicit form.

(3) Find the solution of the recursive relation $a^n = 6a_{n-1} - 9a_{n-2}$, $a_0 = 1$, $a_1 = 6$.

Soln:-

Given! $a^n = 6a_{n-1} - 9a_{n-2}$ —(1), $a_0 = 1$, $a_1 = 6$;

characteristic eqn is.

$r^2 - 6r + 9 = 0$

$\Rightarrow r^2 - 3r - 3r + 9 = 0$

$\Rightarrow r(r-3) - 3(r-3) = 0$

$\Rightarrow (r-3)(r-3) = 0$

$\Rightarrow (r-3)^2 = 0$

$\Rightarrow r - 3 = 0$

$\Rightarrow \boxed{r = 3}$ only one root, roots are same, not distinct

$\boxed{r_0 = 3}$

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$a_1 = 6$.

solution of (1) is given by :

$$a_n = \alpha_1 r_0^n + n\alpha_2 r_0^n$$

$$\Rightarrow a_n = \alpha(3)^n + n\alpha_2(3)^n \quad \text{— (2)}$$

Given that $a_0 = 1$, $a_1 = 6$

Put $n=0$ in (2)

$$a_0 = \alpha_1(3)^0 + 0\alpha_2(3)^0$$

$$1 = \alpha_1(1) + 0$$

$$\therefore \boxed{\alpha_1 = 1}$$

Put $n=1$ in (2)

$$a_1 = \alpha_1(3)^1 + (1)\alpha_2(3)^1$$

$$\Rightarrow 6 = 3\alpha_1 + 3\alpha_2$$

$$\Rightarrow 6 = 3(1) + 3\alpha_2$$

$$\Rightarrow 6-3 = 3\alpha_2$$

$$3 = 3\alpha_2$$

$$\therefore \boxed{\alpha_2 = 1}$$

Substitute $\alpha_1 = 1$, $\alpha_2 = 1$ in (2)

$$a_n = (1)(3)^n + n(1)3^n$$

$$\Rightarrow a_n = (1+n)3^n$$

$$\Rightarrow \boxed{a_n = (n+1)3^n}$$ which is the recursive solution.

(14) Solve the recurrance relation:

$a_n = a_{n-1} + 6 \cdot a_{n-2}$ where $a_0 = 3$
$a_1 = 6$

Soln:-

Given, $a_n = a_{n-1} + 6 \cdot a_{n-2}$, where $a_0 = 3$, $a_1 = 6$ — (1)

characteristics equation is:

$r^2 - r - 6 = 0$

$\cancel{r^2 - 6r + r - 6 = 0}$

$r(r-3) + 2(r-3) = 0$

$(r+2)(r-3) = r - 3 = 0, \; r + 2 = 0$

$\Rightarrow r = -2 \;, \; r = 3$

roots are distinct.

$\boxed{r_2 = 3, \quad r_1 = -2}$

solution of (1) given by:

$a_n = \alpha_1 r_1^n + \alpha_2 r_2^n$

$\Rightarrow a_n = \alpha_1 (-2)^n + \alpha_2 (3)^n$ — (2)

Given $a_0 = 3$, $a_1 = 6$

- Put $(n=0)$ in (2)

$a_0 = \alpha_1 (-2)^0 + \alpha_2 (3)^0$

$3 = \alpha_1 (1) + \alpha_2 (1)$

$\Rightarrow \alpha_1 + \alpha_2 = 3$ — (3)

- Put $(n=1)$ in (2)

$a_1 = \alpha_1 (-2)^1 + \alpha_2 (3)^1$

$6 = -2\alpha_1 + 3\alpha_2$

$\Rightarrow -2\alpha_1 + 3\alpha_2 = 6$ — (4)

(solve) adding (3) & (4)

$$x_1 + x_2 = 3 \quad \times 2$$

$$2x_1 + 2x_2 = 6 \quad — (3)$$

$$-2x_1 + 3x_2 = 6 \quad — (4)$$

$$5x_2 = 12$$

$$\boxed{x_2 = \frac{12}{5}}$$

- Substitude $x_2 = \frac{12}{5}$ in (3)

$$x_1 + x_2 = 3$$

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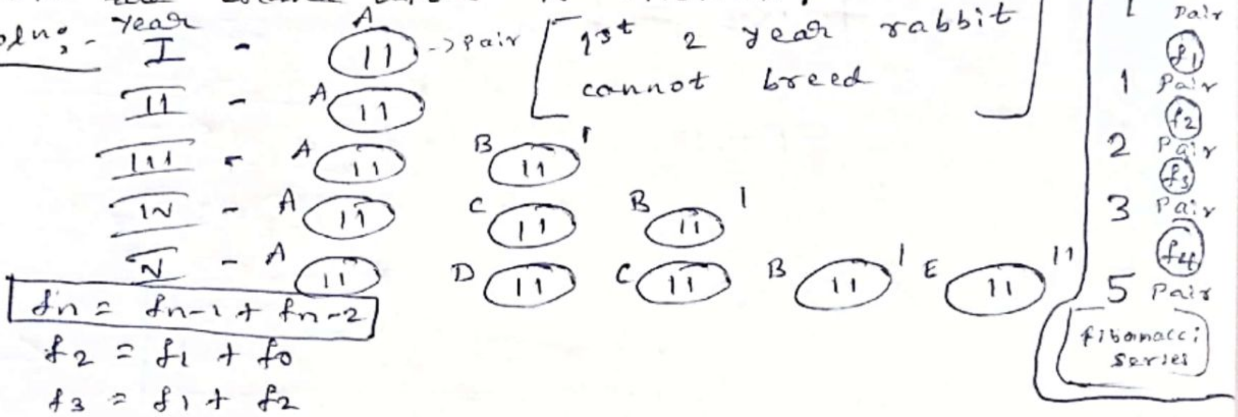
→ Modelling with recurrance relations using Fibonacci rabbits:

① A young pair of rabbits (1 male, 1 female) is placed on an island. the pair cannot breed until they are 2 months old. After they are 2 months old, each pair of rabbits produces another pair each month. Assuming the rabbits never die, Find the recurrance relation for no of rabbits on the island after 'n' months.

Soln:-



$\boxed{f_n = f_{n-1} + f_{n-2}}$

$f_2 = f_1 + f_0$

$f_3 = f_1 + f_2$

$f_4 = f_3 + f_2$

Step-1:- In 1st month 2 rabbits cannot breed.

Step-2:- In 2nd month again 2 rabbits cannot breed as they are young.

Step-3:- In 3rd month these 2 rabbits creates anoth 2 rabbits. Now Total - 2 pairs.

Step-4:- In month-4 old rabbits create another 2 rabbits & new born rabbits 1-month old now Now there are 3-Pairs,

Step-5:- In 5th month there are 4 old rabbits now they create another 4 rabbits & another 2 rabbits born on 4th month become 1-month old now,

There fore, there are totally 10 rabbits i,e, 5 pairs.

Month	Rabbits in Pairs	
I	1	f_0
II	1	f_1
III	2	f_2
IV	3	f_3
V	5	f_4

- By data we get
$f_2 = f_1 + f_0$
$f_3 = f_1 + f_2$
$f_4 = f_3 + f_2$
= = =

$\boxed{f_n = f_{n-1} + f_{n-2}}$ where $f_0 = 1, f_1 = 1$

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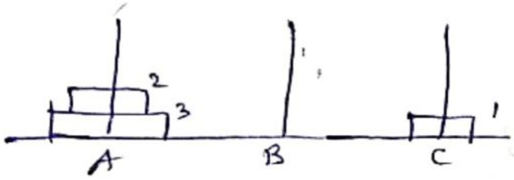
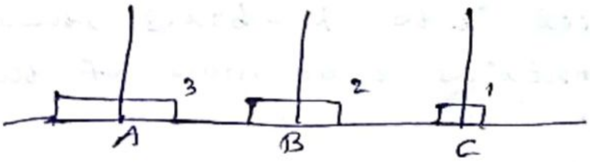
↳ Modelling with recurrance relations using Tower of hanoi Problem:-

- The tower of hanoi is a mathematical problem which consists of 3 rod & multiple disks initially.
- All the disks are placed on 1-rod, 1 cover the other in ascending order of size similar to a cone shaped tower.
- The objective of this problem is to move the disk from initial rod to another rod.

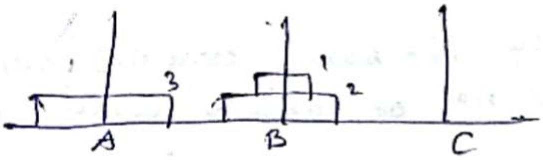
step-1:- Move ~~grey~~ disk-1 from rod A to C



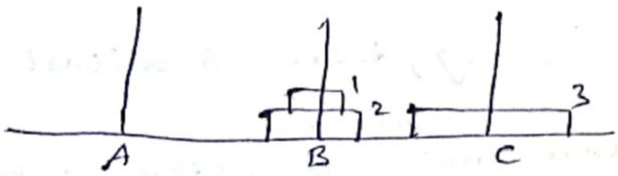
step-2:- move disk-2 from rod A to B



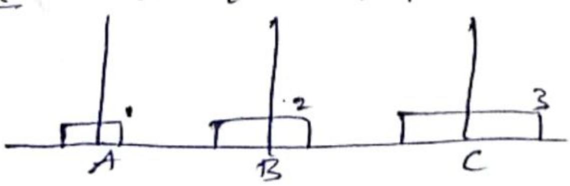
step-3:- move disk-1 from C to A



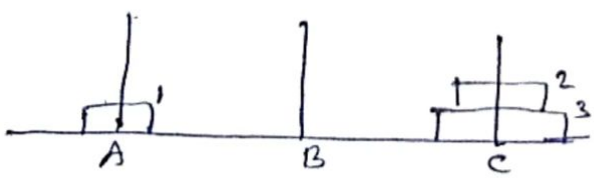
step-4:- Move disk-3 from A to C



step-5:- move disk-1 from B to A



step-6:- move disk-2 from B to C



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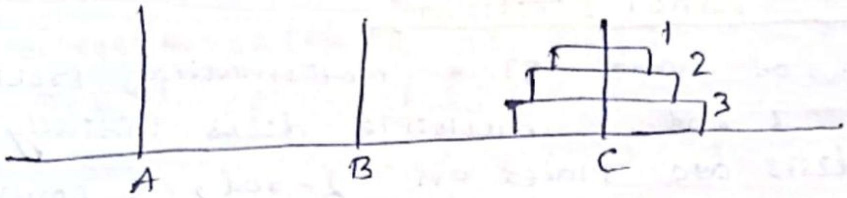
Step-7 :- Move disk-1 from A to C



∴ The puzzle is finally completed, we moved all the 3-disks from rod A to C using 7-steps, ~~ie~~ 2

- Let Hn denotes no of moves required to solve puzzle with 'n' disks.
- Now we can define Hn recursively as :

$\boxed{H_n = 2H_{n-1} + 1}$ where $H_1 = 1$

$$H_3 = 2H_{3-1} + 1$$
$$= 2H_2 - 1 + 1$$
$$= 2(2) - 1 + 1$$

↳ Pigeonhole principle :-

Statement :- If K is a positive integer & K+1 is more objects are placed into K-boxes, then there is atleast 1-box contains 2 or more of the objects.

Proof :- we will prove the pigeonhole principle using contradiction method.

- Suppose that none of the K-boxes contains more than 1-objects. Then total no of objects would be at-most 'K'.
- This is the contradiction because there are atleast K+1 objects.
- Hence our assumption is wrong, there is atleast 1-box containing 2-or-more objects.

Eg:- out of 367-people there must be atleast 2 with the same birthday because there are only 366 possible birthdays.

↳ Representing Relations :-

* Representing Relations using matrices :-

A relation b/w finit sets can be represented using [0,1] matrix.

Suppose $R \xrightarrow{\text{relation from}} A$ to B then relation 'R' can be represented by the matrix $M_R = [M_{ij}]$ where

$$M_{ij} = \begin{cases} 1 & \text{if } (a,b) \in R \\ 0, & \text{if } (a,b) \notin R \end{cases}$$

Eg:- Let $A = \{1, 2, 3\}$, $B = \{1, 2\}$ R is a relation from $A \to B$ defined by : $R = \{(a,b), a \in A, b \in B$ $a > b\}$ write matrix representation of R.

$R = \{(\underset{\text{row}}{2}, \underset{\text{col}}{1}), (\underset{\text{row}}{3}, \underset{\text{col}}{1}), (\underset{\text{row}}{3}, \underset{\text{col}}{2})\}$

$$M_R = \begin{array}{c c} & \begin{array}{cc} 1 & 2 \end{array} \\ \begin{array}{c} 1 \\ 2 \\ 3 \end{array} & \begin{bmatrix} 0 & 0 \\ 1 & 0 \\ 1 & 1 \end{bmatrix} \end{array} \quad 3 \times 2$$

$A = \{1, 2, 4, 5\}$

Eg:- Let $A = \{1, 2, 4, 5\}$ & R is a relation on set A defined by $R = \{(a,b) : a, b \in A$ $a \le b\}$

$R = \{(1,1), (1,2), (1,4), (1,5), (2,2), (2,4), (2,5)$
$(4,4), (4,5), (5,5)\}$

$$M_R = \begin{array}{c c} & \begin{array}{cccc} 1 & 2 & 4 & 5 \end{array} \\ \begin{array}{c} 1 \\ 2 \\ 4 \\ 5 \end{array} & \begin{bmatrix} 1 & 1 & 1 & 1 \\ 0 & 1 & 1 & 1 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 \end{bmatrix} \end{array} \quad 4 \times 4$$

Eg:- Let $A = \{a_1, a_2, a_3\}$ $B = \{b_1, b_2, b_3, b_4, b_5\}$ which added pairs are in the relation R represented by the matrix :

$$M_R = \begin{array}{c c} & \begin{array}{ccccc} b_1 & b_2 & b_3 & b_4 & b_5 \end{array} \\ \begin{array}{c} a_1 \\ a_2 \\ a_3 \end{array} & \begin{bmatrix} 0 & 1 & 0 & 0 & 0 \\ 1 & 0 & 1 & 1 & 0 \\ 1 & 0 & 1 & 0 & 1 \end{bmatrix} \end{array}$$

$R = \{(a_1, b_2), (a_2, b_1), (a_2, b_3), (a_2, b_4),$
$(a_3, b_1), (a_3, b_3), (a_3, b_5)\}$

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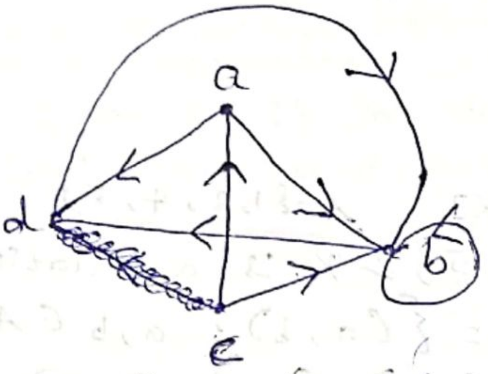
* Representing Relations using Diagraphs :-

- A directed graph or diagraph consists of set of 'V' of vertices together with a set 'E' of ordered pairs of elements of V called edges. The vertex 'a' is called initial vertex of edge (a, b) & vertex 'b' is called terminal vertex of the edge.
- An edge of the form (a, a) is represented using an 'arc' from a vertex 'a' back to itself. Such edge is called a loop.

Eg:- Let $A = \{a, b, c, d\}$ R is a relation on set A defined by, $R = \{(a, b), (a, d), (b, b), (b, d), (c, a), (c, b), (d, b)\}$

write diagraph of 'R'.



↳ Partial ordering:- A relation 'R' on a set 'A' is called partial ordering if it is reflexive, transitive & anti-symmetric.

- A set 'A' together with partial ordering 'R' is called partially ordered set or po-set & is denoted by (A, R).
- Anti-symmetric :

$(a, b) \in R$, then $(b, a) \in R$

$(a, b)(b, a) \in R$, then $a = b$

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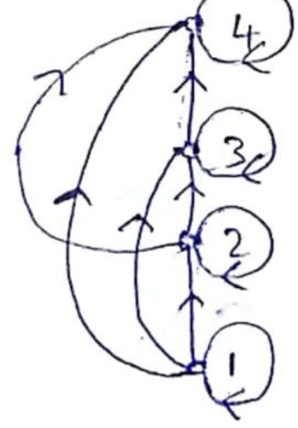
→ **Hasse diagram:-** In general we can represent a partial ordering on a finite set using the following procedure:

- start with a directed graph for given relation.
- because partial ordering is reflexive, a loop is present at every vertex. Remove these loops.
- Remove all edges that must be in the partial ordering because of presence of other edges & transitivity.
- Finally arrange each edge so that its initial vertex is below its terminal vertex.
- Remove all arrows on directed edges because all edges point upward toward their terminal vertex.
- These steps are well-defined & only a finite no of steps need to be carried out for a finite po-set.
- when all the steps have been taken the resulting diagram contains sufficient information to find the partial ordering. This diagram is called Hasse diagram.

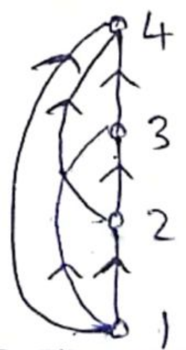
Eg:- $A = \{1, 2, 3, 4\}$ R is relation on set 'A'

$R = \{(a, b) : a, b \in A, a \le b\}$ write Hasse diagram for 'R'.

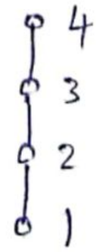
$R = \{(1, 1), (1, 2), (1, 3), (1, 4)$
$(2, 2), (2, 3), (2, 4)$
$(3, 3), (3, 4), (4, 4)\}$



Remove all loops:



Hasse diagram (Remove transitive edges & arrows):



② Draw the Hasse diagram representing the partial ordering (Poset):

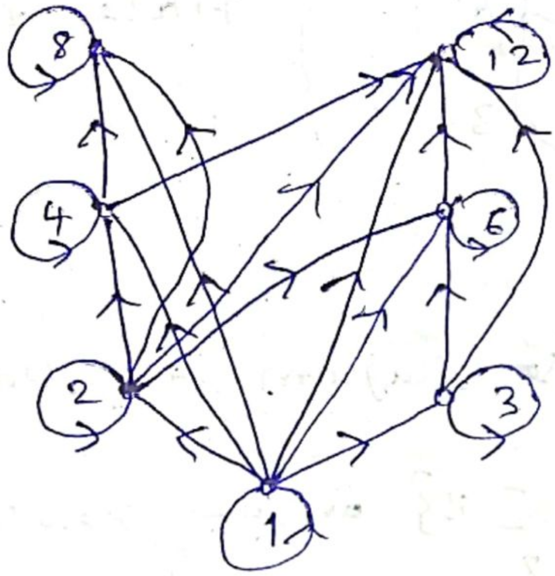
set {(a, b): a divides b} on {1, 2, 3, 4, 6, 8, 12}

Soln:- Given A = {1, 2, 3, 4, 6, 8, 12}

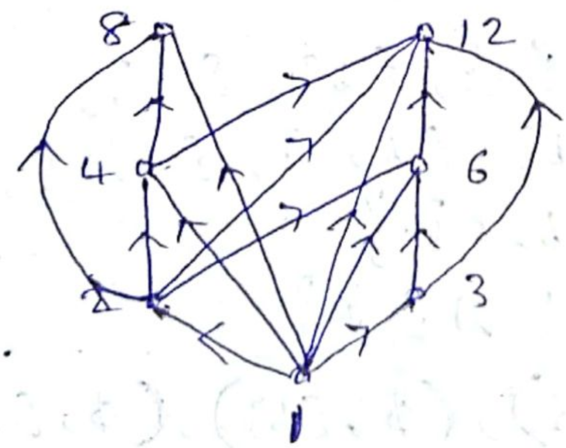
A = {1, 2, 3, 4, 6, 8, 12}

R = {(a, b) : a divides b}

R = {(1, 1) (1, 2) (1, 3) (1, 4) (1, 6) (1, 8) (1, 12),
(2, 2) (2, 4) (2, 6) (2, 8) (2, 12),
(3, 3) (3, 6), (3, 12),
(4, 4) (4, 8) (4, 12),
(6, 6) (6, 12),
(8, 8) (12, 12) }



• Removing loop :-



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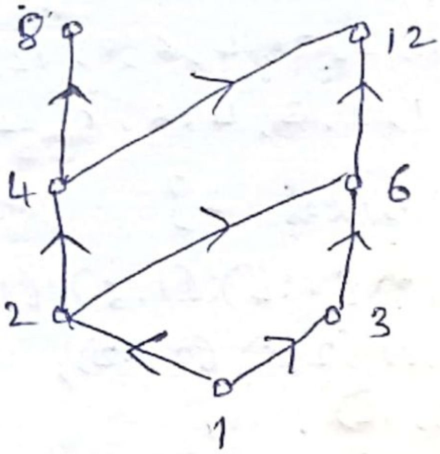
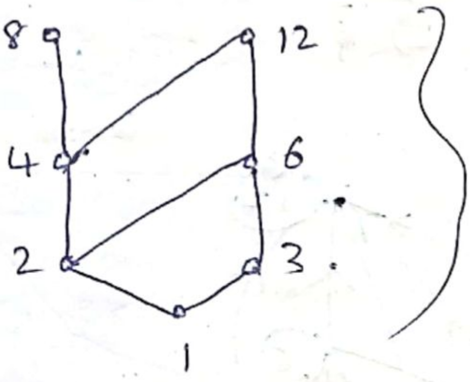
- After removing all transitives Property:

(1, 4) (1, 8) (1, 6) (1, 12) (3, 12) (2, 8) (2, 12)



- After removing all arrows:



Hasse diagram

③ Draw the Hasse diagram for partial ordering:
set $\{(A, B) : A \subseteq B\}$ on the power set $P(S)$
where $S = \{a, b, c\}$

Soln:- Given $S = \{a, b, c\}$

$P(S) = \{\phi, \{a\}, \{b\}, \{c\},$
$\{a, b\}, \{b, c\}, \{a, c\},$
$\{a, b, c\}\}$

$R = \{(A, B) : A \subseteq B\}$

$(\phi, \{a\}), (\phi, \{b\}), (\phi, \{c\})$
$(\phi, \{a, b\}), (\phi, \{b, c\}), (\phi, \{a, c\})$
$(\phi, \{a, b, c\})$ — — —

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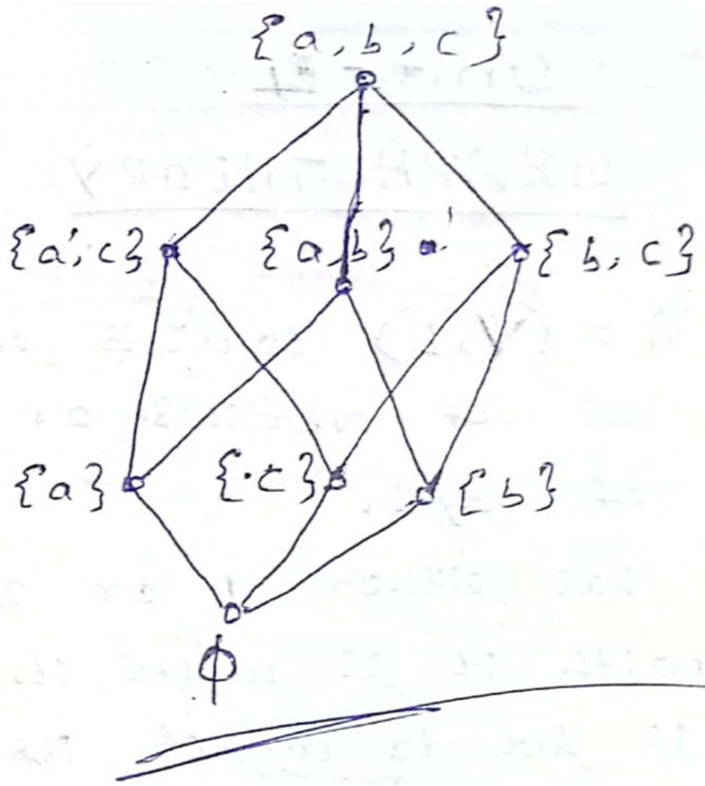

{a, b, c}
{a, c}
{a, b}
{b, c}
{a}
{c}
{b}
ϕ

Unit - 4

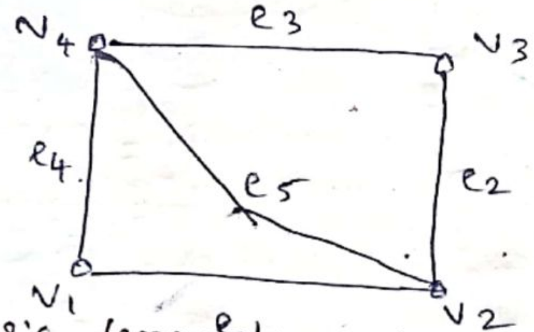
GRAPH THEORY

⟹ Graph :-

- A graph $G = (V, E)$ consists of : (V) - A non-empty set of vertices or nodes & (E) - A set of edges.
- Each edge has either 1 or 2 vertices associated with it is called its end points. An edge is set to connect its end-points.

Eg:- $V = \{v_1, v_2, v_3, v_4\}$

$E = \{e_1, e_2, e_3, e_4, e_5\}$



↳ Basic terminologies :-

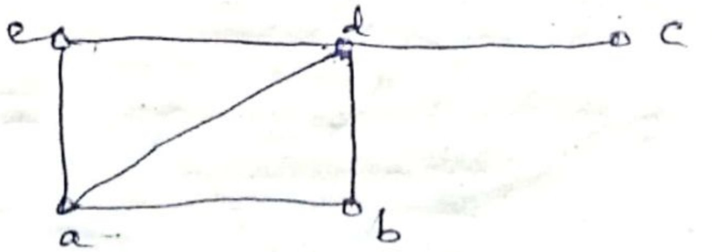
*** Finite graph :-**

- A graph with a finite vertex set is called finite graph.

*** Infinite graph :-** A graph with an infinite vertex set is called infinite graph.

*** Simple graph :-** A graph which (with) has no multiple edges & no loops.

Eg:- $G =$



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~~Basic~~

* **LOOP :-** A loop is an edge that connects a vertex to itself.

Eg :- 

* **Adjascent & Incident vertices :-**

- vertices v_1 & v_2 are adjascent vertices if & only if (v_1, v_2) is an edge in the graph. (or)
- so 2 vertices (u, v) in an undirected graph G are called adjascent in G if u & v are end-points of an edge of G.
- If e is associated with (u, v) the edge 'e' is called Incident with the vertices u & v.

* **Degree of vertices :-** The degree of a vertex in an undirected graph is the no of edges incident with it, except that a loop at a vertex contributes twice to the degree of that vertex.

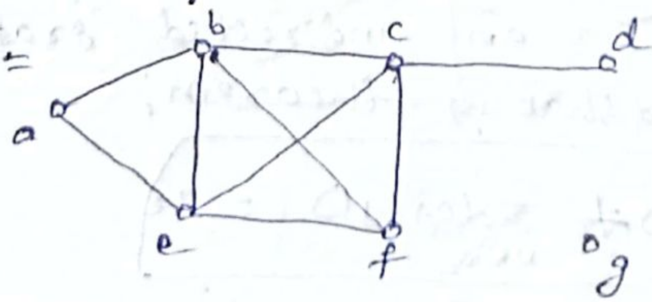
• The degree of the vertex is denoted by : $\boxed{\deg(\,)}$

Note :-

• A vertex of degree '0' is called isolated vertex.

• A vertex is pendent if & only if it has degree '1'

• Eg :- G =



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$deg(a) = 2$
$deg(b) = 4$
$deg(c) = 4$
$deg(d) = 1$
$deg(e) = 4$
$deg(f) = 3$
$deg(g) = 0$

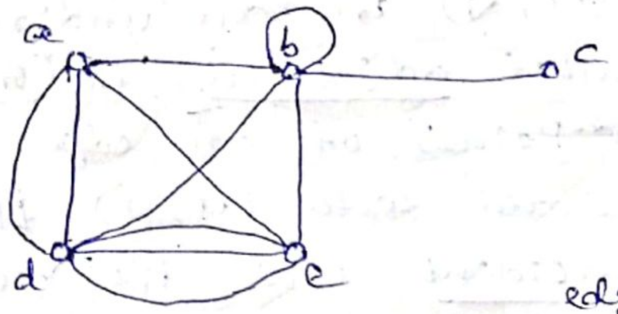
$\therefore$ The total should be in <u>Even</u>

edges $= 9$

$9(2) = 18$

<u>Eg:-</u>

$H =$



$deg(a) = 4$ even
$deg(b) = 6$ even
$deg(c) = 1$ odd
$deg(d) = 6$ even
$deg(e) = 5$ odd

edges $= 11$

$11(2) = 22$

$V_1 = \{c, e\}$ set of odd degree vertices
$V_2 = \{a, b, d\}$ set of even degree vertices

Handshaking theorem:-

- Let $G = (V, E)$ be an undirected graph with 'e'- no of edges than $\boxed{\sum_{u \in V} deg(u) = 2e}$

Theorem:- An undirected graph has an even no of vertices of odd degree.

Proof:- Let V_1 & V_2 be the set of vertices of even degree & set of vertices of odd degree respectively. In an undirected graph $G = (P, E)$

$\therefore$ By handshaking theorem;

$$\boxed{\sum_{u \in V_1} deg(u) + \sum_{u \in V_2} deg(u) = 2e}$$

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- Here V_1 is a set of even degree vert therefore:

$\sum_{u \in V_1} \deg(u)$ is even no.

- Hence $\boxed{\sum_{u \in V_2} \deg(u)}$ must be an even no.

$\therefore$ It is clear that an undirected graph has an even no of vertices of odd degree.

→ Directed graph (Diagraph) :-

- A directed graph (V, E) consists of a non-empty set of vertices (V) & set of directed edges (E).
- Each directed edge is associated with a ~~ordered~~ ordered pair of vertices.
- The directed edge associated with added pair (u, v) is said to start at 'u' & end at 'v'.

$$u \xrightarrow{e} v$$

↳ ~~Degree of directed graph :-~~ Definition :-

- When (u, v) is an edge of the graph 'G' with directed edges, u is said to be adjacent to v & v is said to be adjacent from u.

$$u \longrightarrow v$$

- The vertex u is called the initial vertex of edge (u, v) & v is called terminal vertex.

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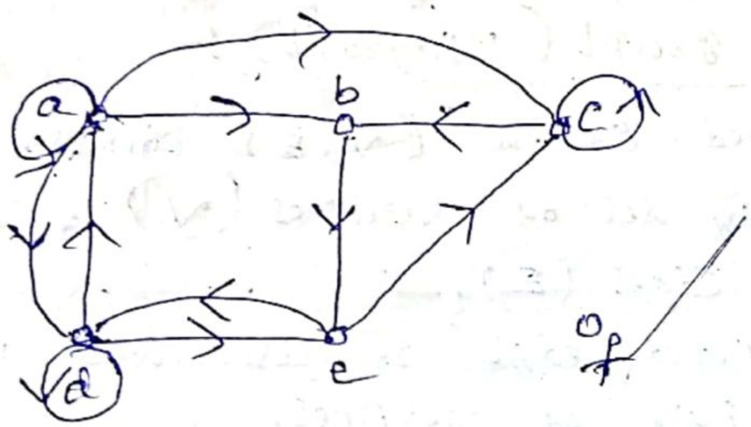
↳ Degrees of directed graph :-

- In a graph with directed edges, the indegree of vertex (v) denoted by : $\boxed{deg^-(v)}$ is the no of edges with (v) as their terminal vertex.
- The outdegree of vertex (v) denoted by ; $\boxed{deg^+(v)}$ is the no of edges with (v) as their initial vertex.

Eg:-

G =



Inorder :-

$deg^-(a) = 2 \longrightarrow$ a → a, b → a

$deg^-(b) = 2 \longrightarrow$ a → b, c → b

$deg^-(c) = 3 \longrightarrow$ c → a, e → c, a → c

$deg^-(d) = 3 \longrightarrow$ a → d, e → d, d → d

$deg^-(e) = 2 \longrightarrow$ d → e, b → e

$deg^-(f) = 0$

outdegree :-

$deg^+(a) = 4$ → $a \to a$, $a \to c$, $a \to b$, $a \to d$

$deg^+(b) = 1$ → $b \to e$

$deg^+(c) = 2$ → $c \to c$, $c \to b$

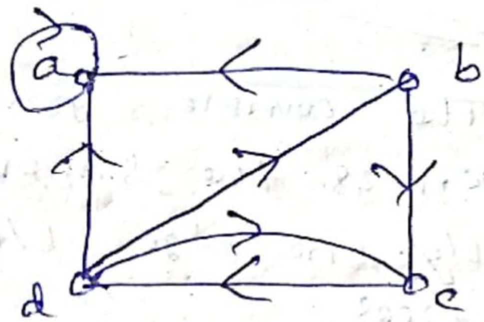
$deg^+(d) = 3$ → $d \to d$, $d \to a$, $d \to e$

$deg^+(e) = 2$ → $e \to d$, $e \to c$

$deg^+(f) = 0$

eg:-

G =



Indegree :-	outdegree :-
$deg^-(a) = 3$	$deg^+(a) = 1$
$deg^-(b) = 1$	$deg^+(b) = 2$
$deg^-(c) = 2$	$deg^+(c) = 1$
$deg^-(d) = 1$	$deg^+(d) = 3$

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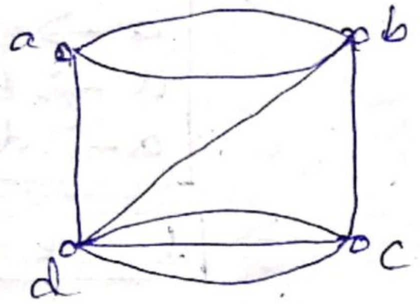
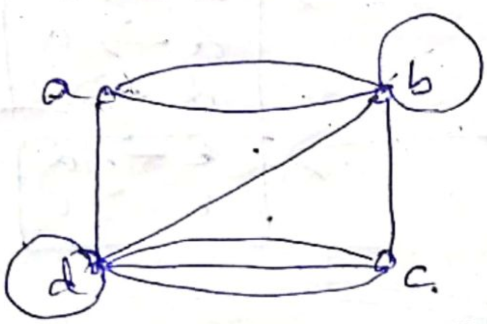
* **Multi graph :-** A graph with multiple edges is called multi-graph.

Eg:-



* **Pseudograph :-** A graph with both multiple edges & loops is called pseudo graph.

Eg:-

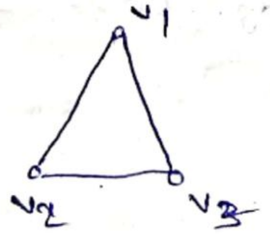


* **Complete graph :-** The comple graph on 'n' values denoted by K_n is the simple graph that contains exactly one edge b/w each pair of distinct vertices.

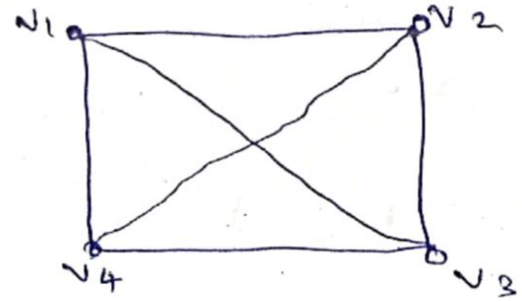
Eg:- K_1 :

K_2 :

K_3 :



K_4 :



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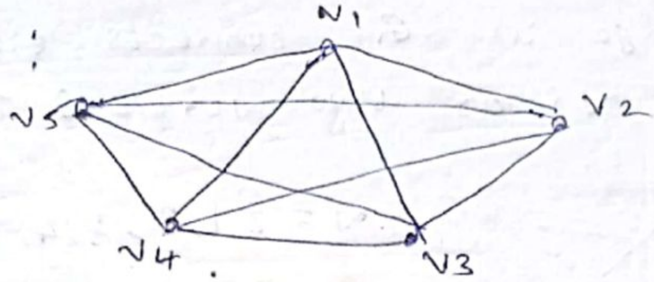
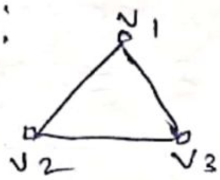
K_5 :

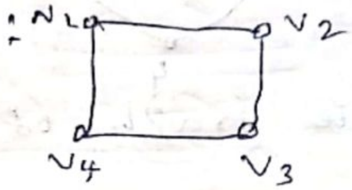


* **Cycle :-** The cycle C_n, $n \geq 3$ consists of 'n' vertices 1, 2, 3, -- n & edges (1, 2), (2, 3), (3, 4) -- -- (n-1, n), (n, 1)

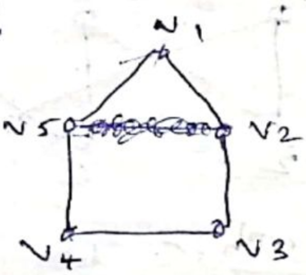
Eg :- C_3 :



C_4 :

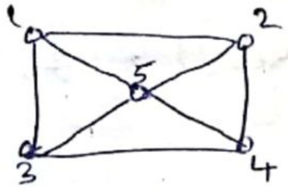


C_5 :

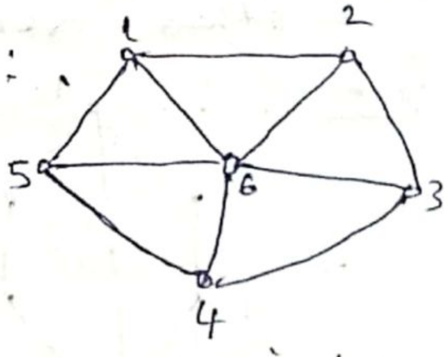


* **Wheels :-** We obtain the wheel 'W_n'. when we add an addition vertex to the cycle 'C_n' for $n \geq 3$ & connect this new vertex ~~to~~ each of the 'n' vertices in 'C_n' by new edges.

Eg :- W_4 :



W_5 :



* **Bipartite graph :-** A simple graph 'G' is called Bipartite, if its vertex set 'V' can be partitioned into 2 disjoint sets V_1 & V_2 such that every edge in the graph connects a vertex set in V_1 & vertex set in V_2.

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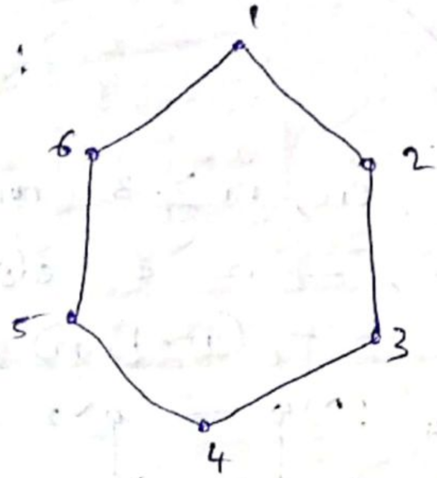
• So that no edge in G connects either 2 vertices in V_1 or two vertices in V_2.

Eg :- G :

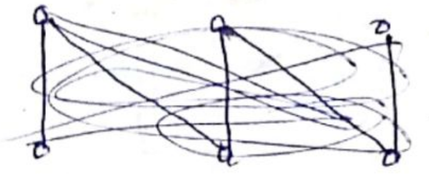


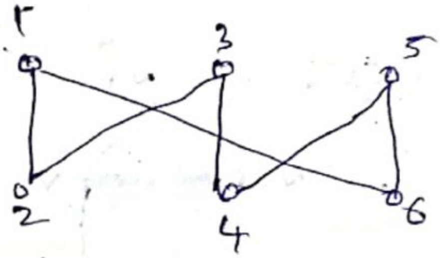
$V = \{1, 2, 3, 4, 5, 6\}$

$V_1 = \{1, 3, 5\}$

$V_2 = \{6, 2, 4\}$

∴ Bipartite graph of G :

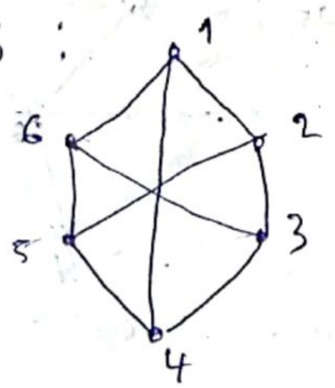




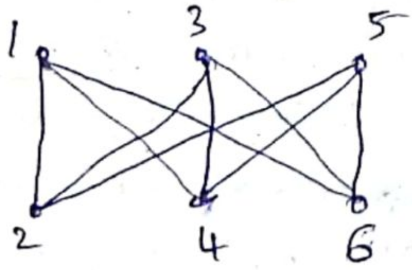
* **Complete Bipartite graph :-** The complete Bipartite graph is the graph that has its vertex is partitioned into 2 subsets of m & n vertices respectively.

• There is an edge b/w 2 vertices if & only if one vertex in the 1st subset & other vertex in the 2nd subset.

Eg :- G :



$K_{3,3}$:



∴ Complete Bipartite graph of G

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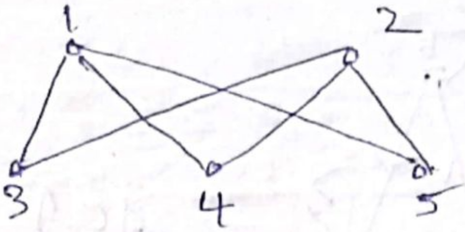
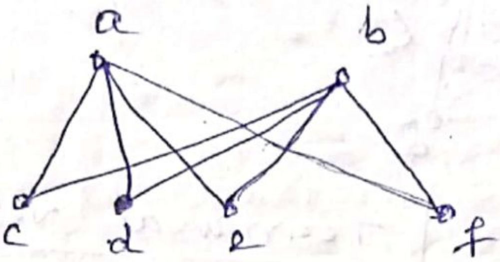
Eg :- $K_{2,3}$:



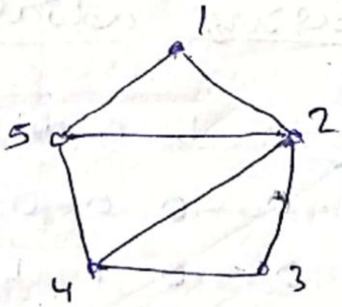
$K_{2,4}$:



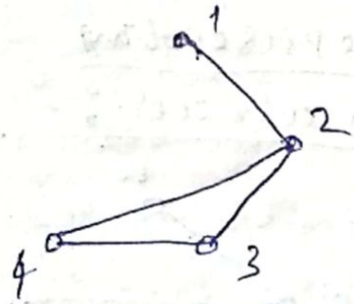
* **Subgraph :-** A subgraph of the graph $G = (V, E)$ is a graph $H = (W, F)$ where $W \subseteq V$ & $F \subseteq E$.

- A subgraph H of G is a proper sub graph of 'G' if $H \neq G$.

Eg :- G :



H :

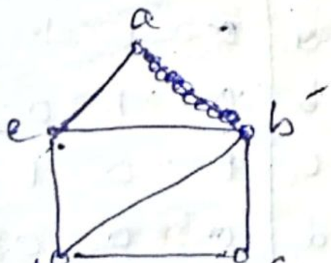


∴ clearly 'H' is subgraph of 'G'.

↳ **Representing graphs :-**

i) **Representing graphs using adjacency list :-**

Eg :- G :



Vertex	Adjacency list
a	e
b	c, d, e
c	b, d
d	b, c, e
e	a, b, d

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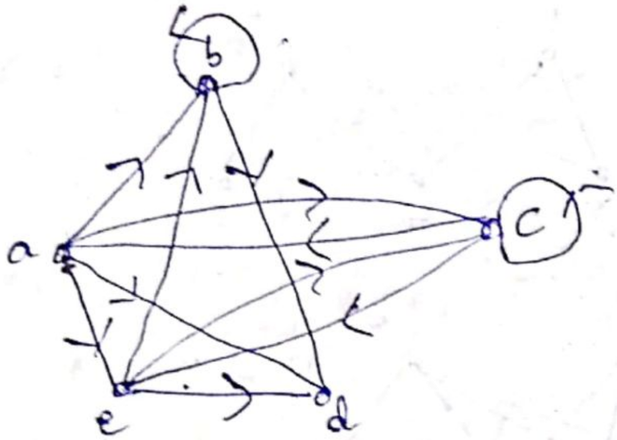
Eg:-

H:



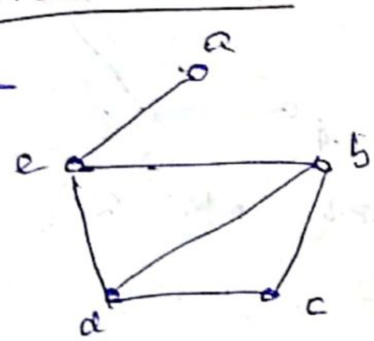
Initial vertex	Terminal vertex
a	b, c, d, e
b	b, d
c	a, c, e
d	—
e	b, c, d

Adjacency list for directed graph

ii) Representing graph using adjacency matrices :-

Eg:-

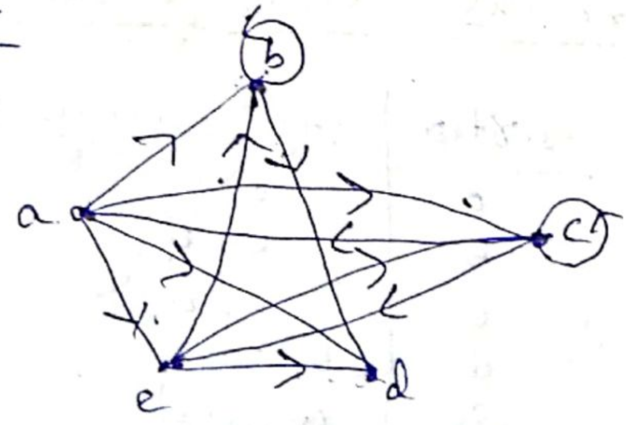
G:



$$A_G = \begin{array}{c c} & \begin{array}{ccccc} a & b & c & d & e \end{array} \\ \begin{array}{c} a \\ b \\ c \\ d \\ e \end{array} & \begin{bmatrix} 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 1 & 1 \\ 0 & 1 & 0 & 1 & 0 \\ 0 & 1 & 1 & 0 & 1 \\ 1 & 1 & 0 & 1 & 0 \end{bmatrix} \end{array}$$

Eg:-

H:



$$A_H = \begin{array}{c c} & \begin{array}{ccccc} a & b & c & d & e \end{array} \\ \begin{array}{c} a \\ b \\ c \\ d \\ e \end{array} & \begin{bmatrix} 0 & 1 & 1 & 1 & 1 \\ 0 & 1 & 0 & 1 & 0 \\ 1 & 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 1 & 1 & 0 \end{bmatrix} \end{array}$$

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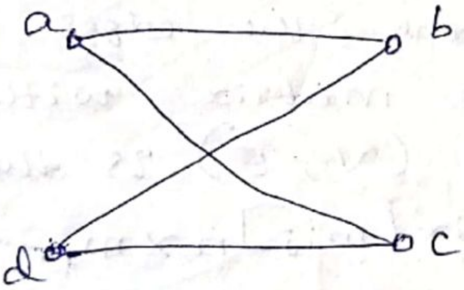
- Suppose $G = (V, E)$ is a simple graph with 'n' no of vertices.
- The adjacency matrix A_G with ~~resting~~ respect to this list of vertices is $n \times n$ with 0, 1 matrix $A_G = [a_{ij}]_{n \times n}$ where,

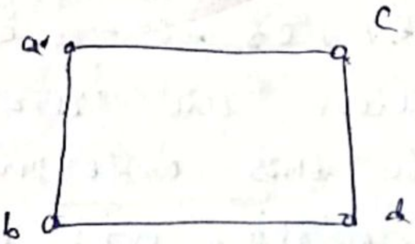
$$a_{ij} = \begin{cases} 1, & \text{if } (i, j) \text{ is an edge of } G \\ 0, & \text{otherwise} \end{cases}$$

Eg:- Draw the graph with adjacency matrix:

$$\begin{array}{c|cccc} & a & b & c & d \\ \hline a & 0 & 1 & 1 & 0 \\ b & 1 & 0 & 0 & 1 \\ c & 1 & 0 & 0 & 1 \\ d & 0 & 1 & 1 & 0 \end{array}$$

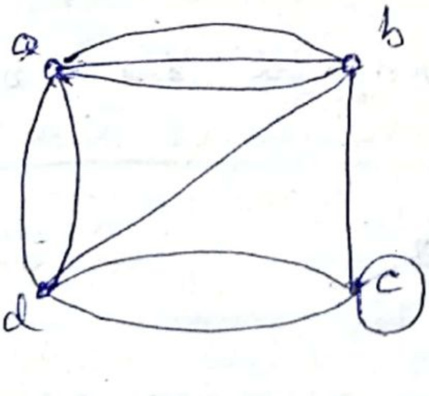


(or)



Eg:- Use an adjacency matrix to represent pseudograph:

G:

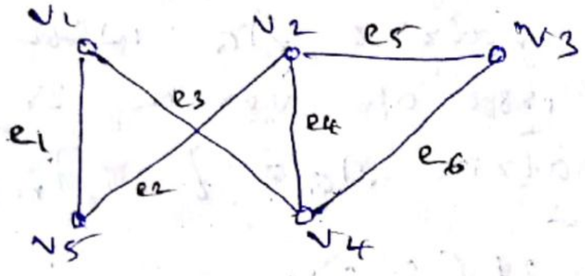


$$A_G = \begin{array}{c} \\ a \\ b \\ c \\ d \end{array} \begin{array}{c} \begin{array}{cccc} a & b & c & d \end{array} \\ \begin{bmatrix} 0 & 3 & 0 & 2 \\ 3 & 0 & 1 & 1 \\ 0 & 1 & 1 & 2 \\ 2 & 1 & 2 & 0 \end{bmatrix} \end{array}$$

$\therefore$ No of edges;

iii) Represent graphs using incidence matrices:

Eg:- G:



The incidence matrices of G:

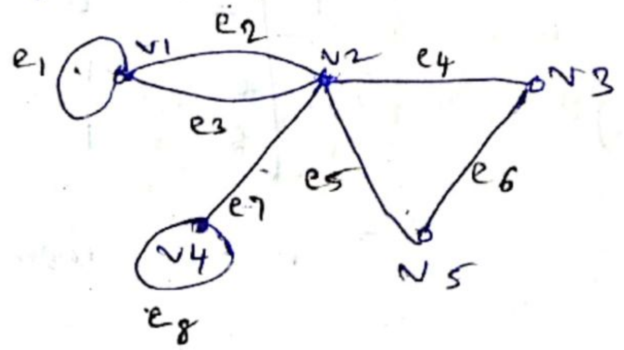
$$M_G = \begin{array}{c} \\ v_1 \\ v_2 \\ v_3 \\ v_4 \\ v_5 \end{array}\begin{array}{c} \begin{array}{cccccc} e_1 & e_2 & e_3 & e_4 & e_5 & e_6 \end{array} \\ \begin{bmatrix} 1 & 0 & 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 & 1 \\ 0 & 0 & 1 & 1 & 0 & 1 \\ 1 & 1 & 0 & 0 & 0 & 0 \end{bmatrix}_{\hat{5} \times \overset{m}{6}} \end{array}$$

- Let $G = (V, E)$ be an undirected graph.
  suppose ~~that~~ $1, 2 - - n$ are vertices &
  $e_1, e_2 - - . e_m$ are the edges of 'G'.
- then the incidence matrix with respect to this ordering of (V, E) is the $n \times m$ matrix, $m =$ matrix $[m_{ij}]_{n \times m}$.

where $m_j = \begin{cases} 1 & \text{when edge } G \text{ is incident with } i \\ 0, & \text{otherwise.} \end{cases}$

Eg:- write incidence matrix for following graph:

G:



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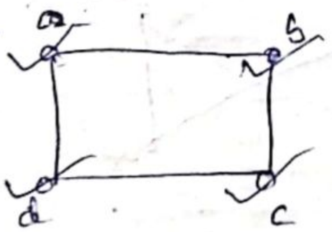
$$M_G = \begin{array}{c} \\ v_1 \\ v_2 \\ v_3 \\ v_4 \\ v_5 \end{array} \begin{array}{c} \begin{array}{cccccccc} e_1 & e_2 & e_3 & e_4 & e_5 & e_6 & e_7 & e_8 \end{array} \\ \begin{bmatrix} 1 & 1 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 1 & 1 & 1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 & 1 & 1 & 0 & 0 \end{bmatrix}_{5\times 8} \end{array}$$

Isomorphism of graphs:-

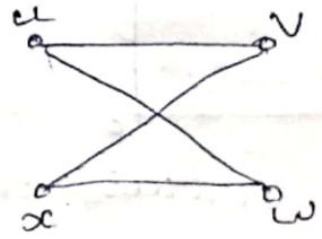
- The simple graphs $G_1 = (V_1, E_1)$ & $G_2 = (V_2, E_2)$ are isomorphic if there is a one-to-one (one-one) & on-to function:
  $f : V_1 \rightarrow V_2$ with the property that 'a' & 'b' are adjascent in G_1 if & only if $f(a)$ & $f(b)$ are adjascent in G_2 $\forall\, a, b \in V_1$
- Such function 'f' is called Isomorphism of graphs.

Eg:-

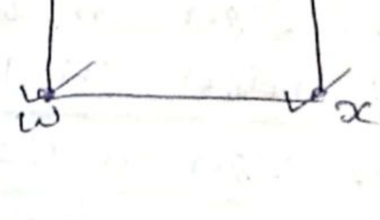
G:



H:



$f(a) = u$
$f(b) = v$
$f(d) = w$
$f(c) = x$



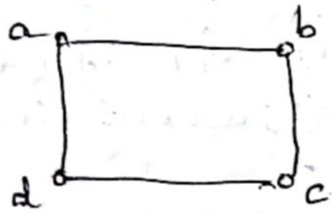
$$A_G = \begin{array}{c} \\ a \\ b \\ c \\ d \end{array} \begin{array}{c} \begin{array}{cccc} a & b & c & d \end{array} \\ \begin{bmatrix} 0 & 1 & 0 & 1 \\ 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \\ 1 & 0 & 1 & 0 \end{bmatrix} \end{array}$$

$$A_H = \begin{array}{c} \\ u \\ v \\ x \\ w \end{array} \begin{array}{c} \begin{array}{cccc} u & v & x & w \end{array} \\ \begin{bmatrix} 0 & 1 & 0 & 1 \\ 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \\ 1 & 0 & 1 & 0 \end{bmatrix} \end{array}$$

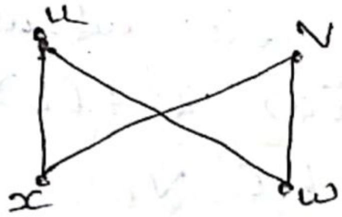
Problems:-

(1) Verify the following graphs are isomorphic or not;

G:

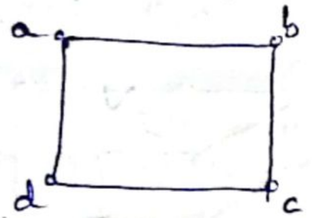


H:



Soln:- Given;

G:



H:

- No of vertices in G = 4
- No of vertices in H = 4

∴ $\boxed{\text{No of vertices in } G = \text{no of vertices in } H}$

- No of edges in G = 4
- No of edges in H = 4

∴ $\boxed{\text{No of edges in } G = \text{no of edges in } H}$

- Degrees of vertices in G = 2, 2, 2, 2
- Degrees of vertices in H = 2, 2, 2, 2

∴ $\boxed{\text{Degrees of vertices in } G = \text{Degrees of vertices in } H}$

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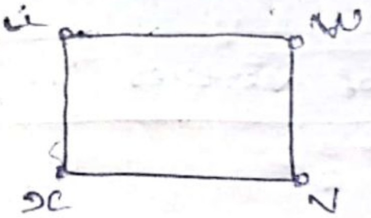
Graph 'H' can be drawn as,



$\therefore f(a) = u$

$f(b) = w$

$f(c) = v$

$f(d) = x$

$\therefore$ There is one-one correspondence b/w vertices of G & H.

- Adjacency matrix of G:

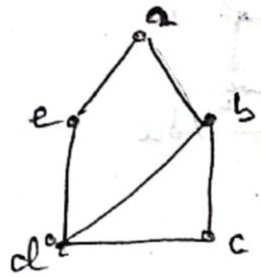
$$A_G = \begin{array}{c} \\ a \\ b \\ c \\ d \end{array} \begin{array}{c} \begin{array}{cccc} a & b & c & d \end{array} \\ \begin{bmatrix} 0 & 1 & 0 & 1 \\ 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \\ 1 & 0 & 1 & 0 \end{bmatrix} \end{array}_{4\times4}$$

- Adjacency matrix of H:

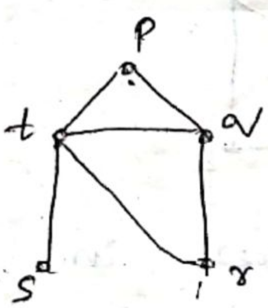
$$A_H = \begin{array}{c} \\ u \\ w \\ v \\ x \end{array} \begin{array}{c} \begin{array}{cccc} u & w & v & x \end{array} \\ \begin{bmatrix} 0 & 1 & 0 & 1 \\ 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \\ 1 & 0 & 1 & 0 \end{bmatrix} \end{array}_{4\times4}$$

(2) Check whether the following graph are isomorphic:

G:

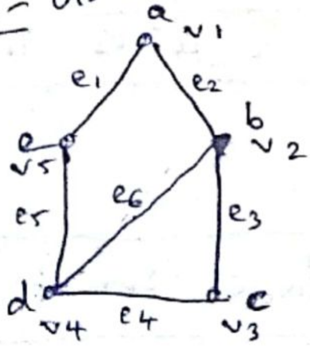


H:

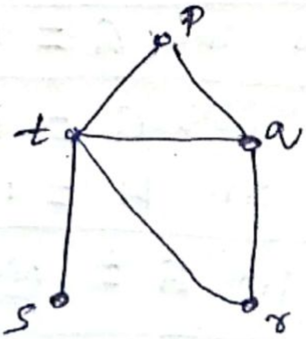


soln:- Given:

G:



H:



- No of vertices in G = 5
- No of vertices in H = 5

∴ No of vertices in G = No of vertices in H

- No of edges in G = 6
- No of edges in H = 6

∴ No of edges in G = No of edges in H

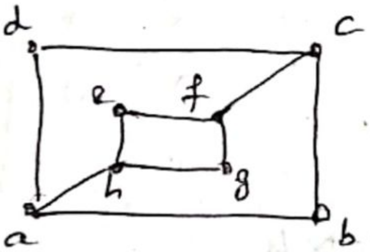
- Degrees of vertices in G = 2, 3, 2, 3, 2
- Degrees of vertices in H = 2, 3, 2, 1, 4

∴ Degrees of vertices in G ≠ Degrees of vertices in H

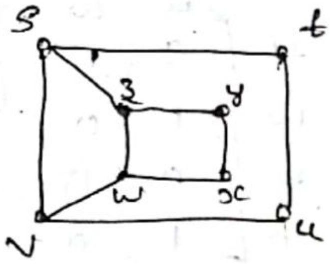
∴ G & H are not isomorphic.

(3) Check whether the following graphs are isomorphic or not;

G:

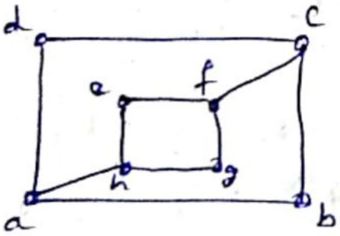


H:

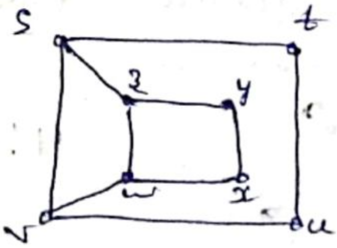


Solⁿ:- Given;

G:



H:



- No of vertices in G = 8
- No of vertices in H = 8

∴ No of vertices in G = No of vertices in H

- No of edges in G = 10
- No of edges in H = 10

∴ No of edges in G = No of edges in H

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- Degrees of vertices in G = 3, 2, 3, 2, 2, 3, 2, 3
- Degrees of vertices in H = 3, 2, 2, 3, 3, 2, 2, 3

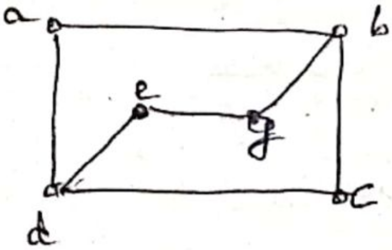
∴ $\boxed{\text{Degrees of vertices in G} = \text{Degrees of vertices in H}}$

∴ Here, graph 'H' has vertex 'W' which is adjacent to with 2 vertices of degree 3. but there is no adjacent. 3.

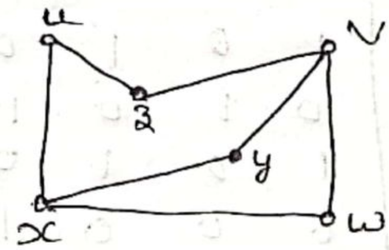
∴ Hence H & G are not isomorphic.

(4) Check whether ~~the~~ following graphs are isomorphic or not :

G:

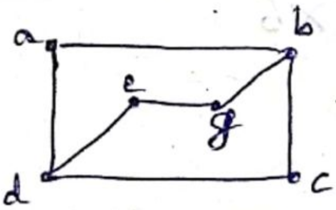


H:

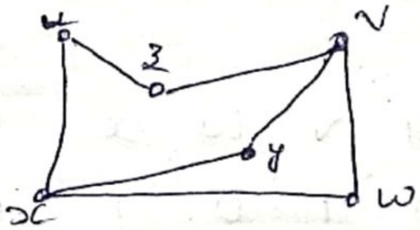


Soln:- Given :

G;



H:



- No of vertices in G = 6
- No of vertices in H = 6

∴ $\boxed{\text{No of vertices in G} = \text{no of vertices in H}}$

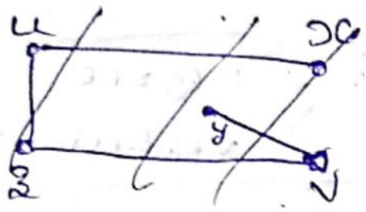
- No of edges in G = 7
- No of edges in H = 7

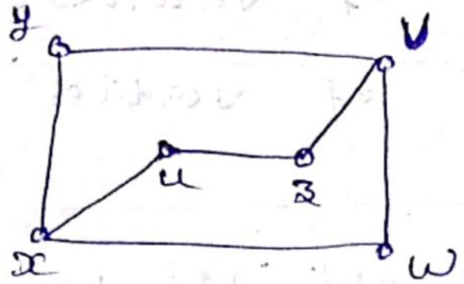
∴ $\boxed{\text{No of edges in G} = \text{No of edges in H.}}$

- Degrees of vertices in G = 2, 3, 2, 3, 2, 2
- Degrees of vertices in H = 2, 3, 2, 3, 2, 2

∴ $\boxed{\text{Degrees of vertices in G} = \text{Degrees of vertices in H.}}$

- The graph 'H' can be drawn as,





$f(a) = y$
$f(b) = v$
$f(c) = w$
$f(d) = x$
$f(e) = u$
$f(g) = z$

∴ There is one-one correspondance b/w vertices of G & H.

- Adjacency matrix of G:

$$A_G = \begin{array}{c} \\ a \\ b \\ c \\ d \\ e \\ g \end{array} \begin{array}{c} \begin{array}{cccccc} a & b & c & d & e & g \end{array} \\ \begin{bmatrix} 0 & 1 & 0 & 1 & 0 & 0 \\ 1 & 0 & 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 1 & 0 & 0 \\ 1 & 0 & 1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 & 0 & 1 \\ 0 & 1 & 0 & 0 & 1 & 0 \end{bmatrix}_{6 \times 6} \end{array}$$

$$A_H = \begin{array}{c} \\ y \\ v \\ w \\ x \\ u \\ z \end{array} \begin{array}{c} \begin{array}{cccccc} y & v & w & x & u & z \end{array} \\ \begin{bmatrix} 0 & 1 & 0 & 1 & 0 & 0 \\ 1 & 0 & 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 1 & 0 & 0 \\ 1 & 0 & 1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 & 0 & 1 \\ 0 & 1 & 0 & 0 & 1 & 0 \end{bmatrix}_{6 \times 6} \end{array}$$

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